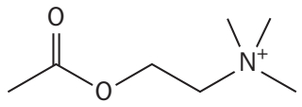
Attractor dynamics

Synaptic Transmission

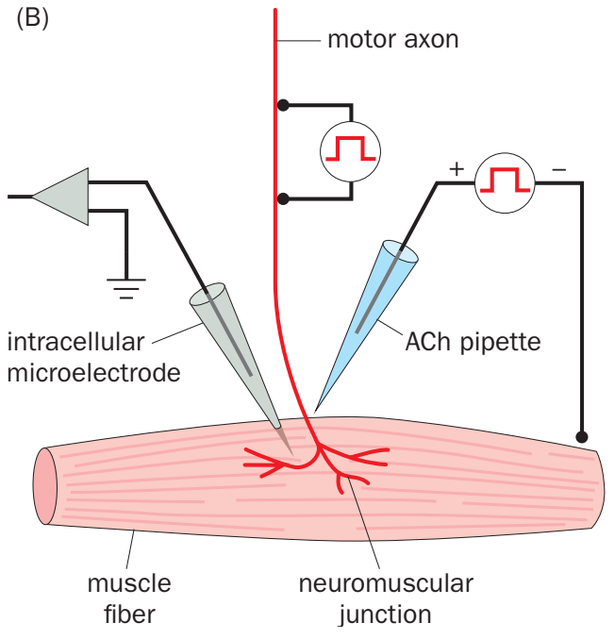
Neurotransmitter release evokes membrane potential change in the postsynaptic neuron

(A)

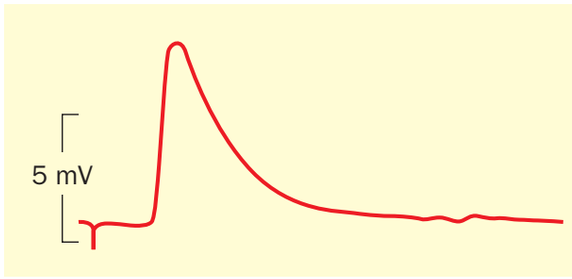


acetylcholine (ACh)

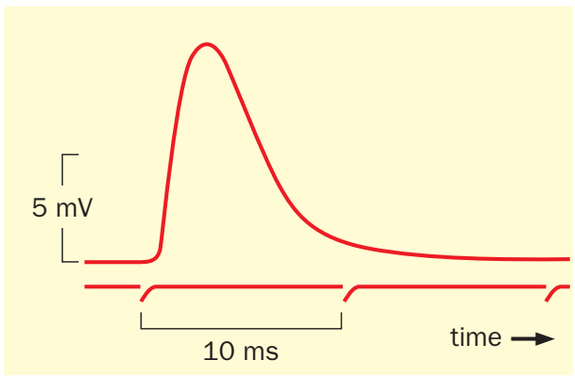
(B)



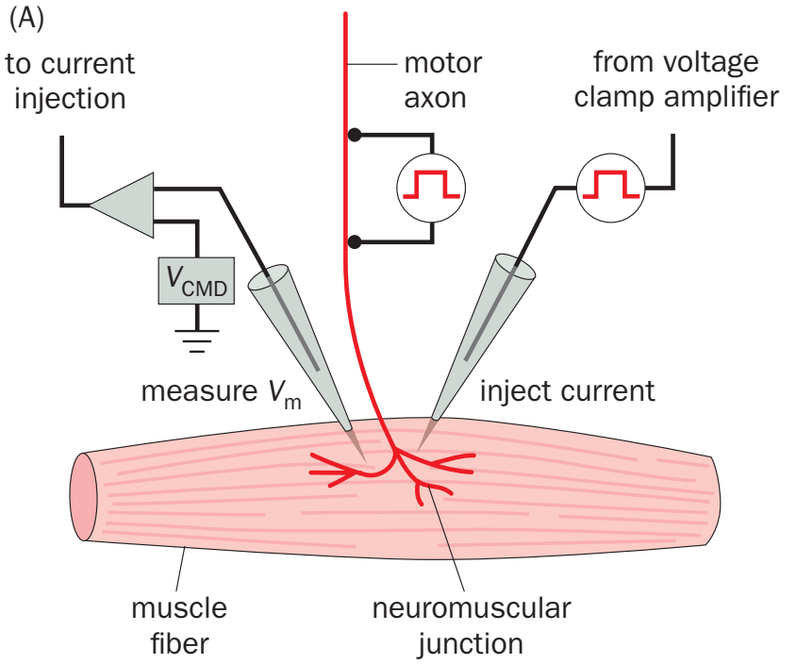
(C) nerve stimulation

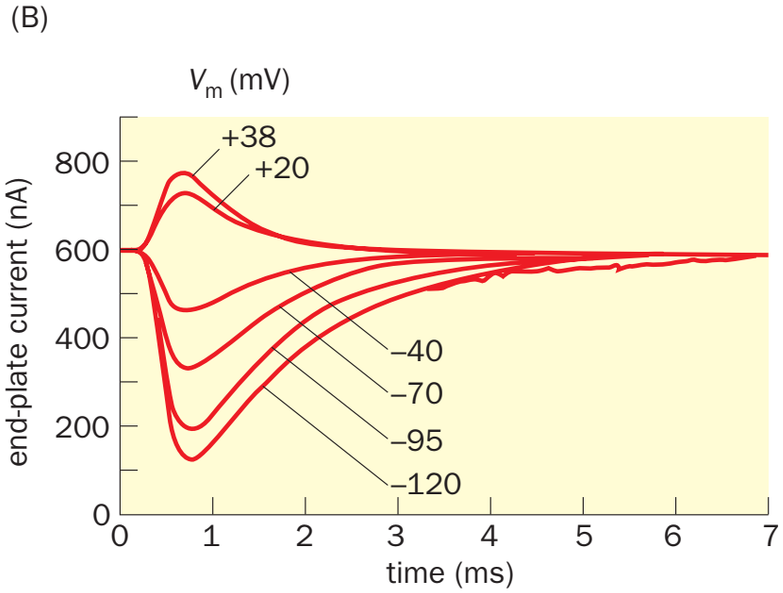


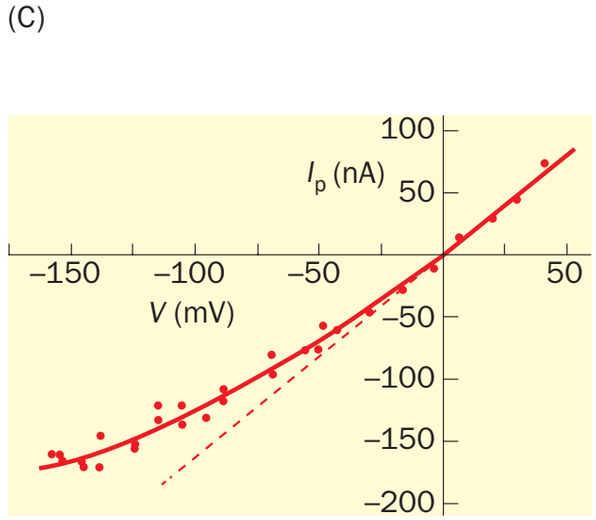
ACh iontophoresis



Reversal potential of a synapse







Modelling synaptic transmission

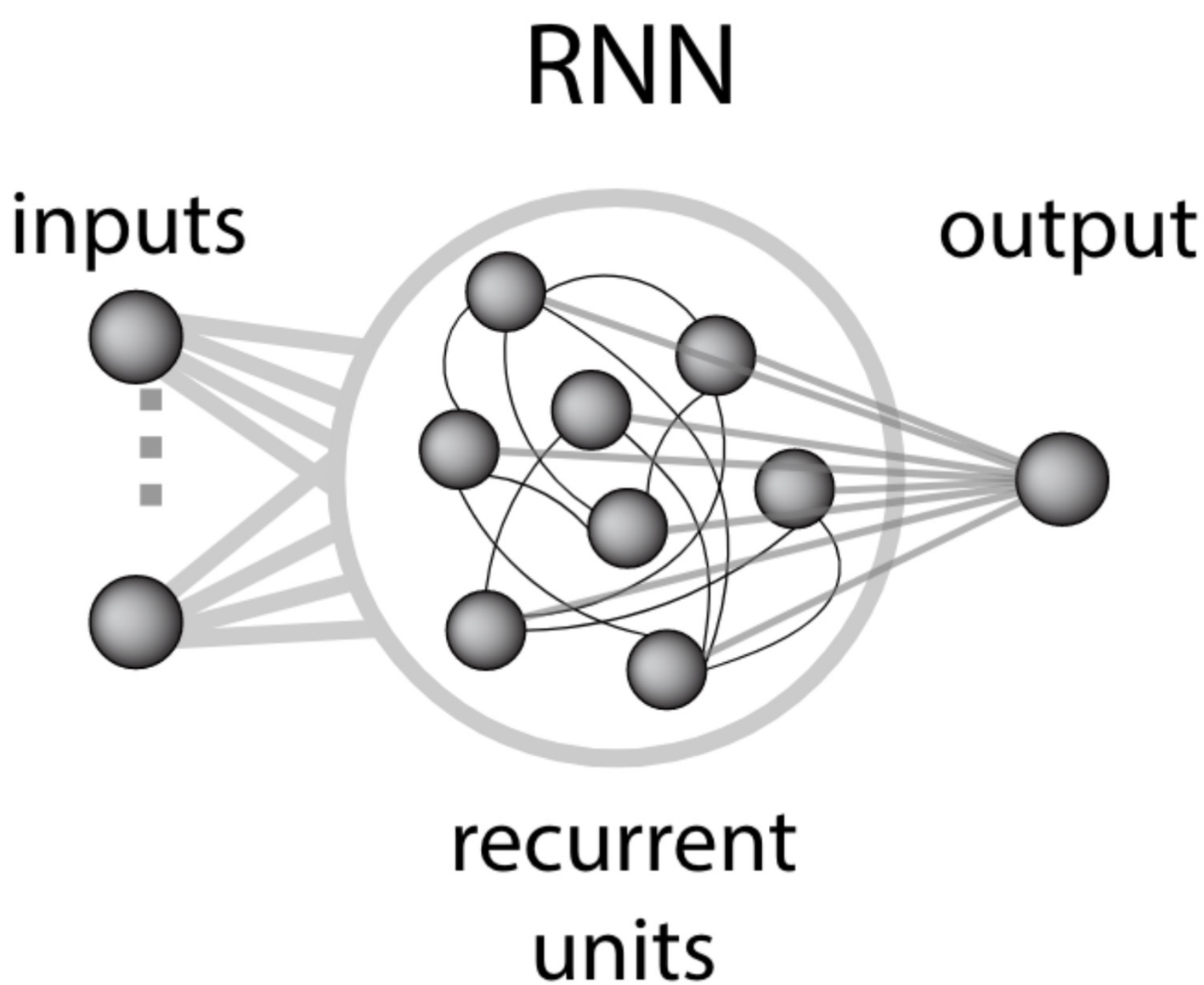
Release probability (u)

$$I = \bar{g}_s u(V - E_s) \approx Au$$

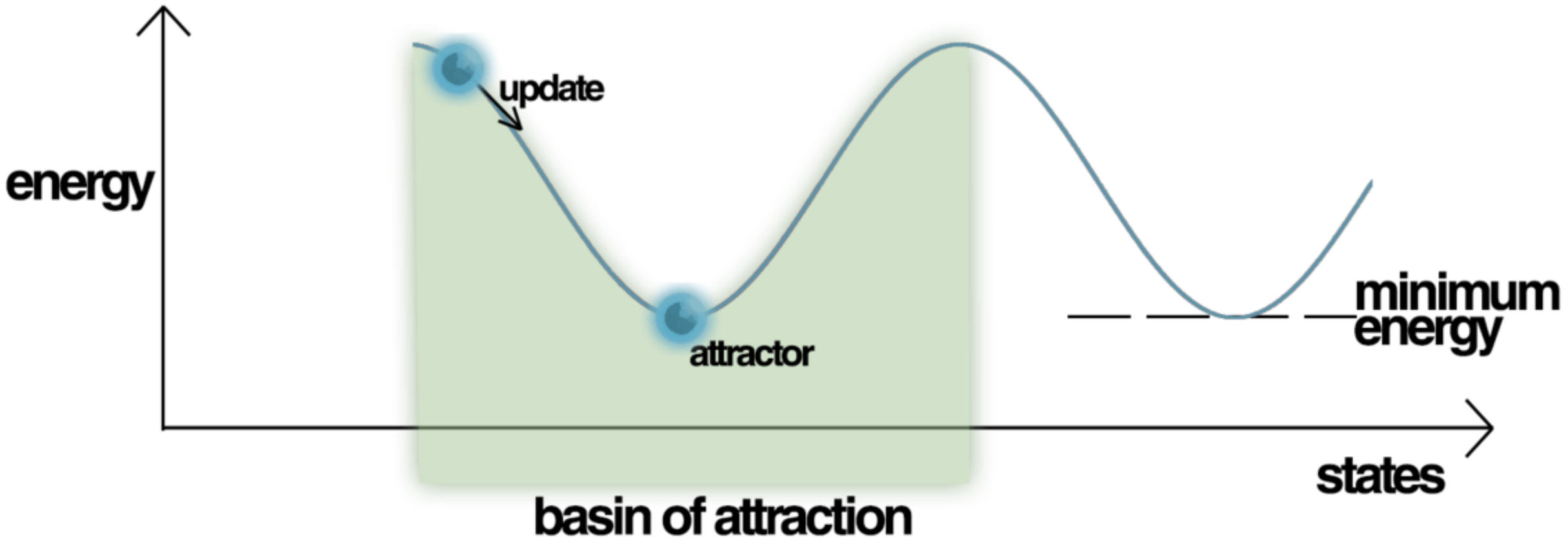
$$u = u_0 e^{-(t-t_{spike})/\tau_s}, \ \textbf{for } t > t_{spike}$$

$$\frac{dI}{dt} = -\frac{I}{\tau_s} + Au_0\delta(t - t_{spike})$$

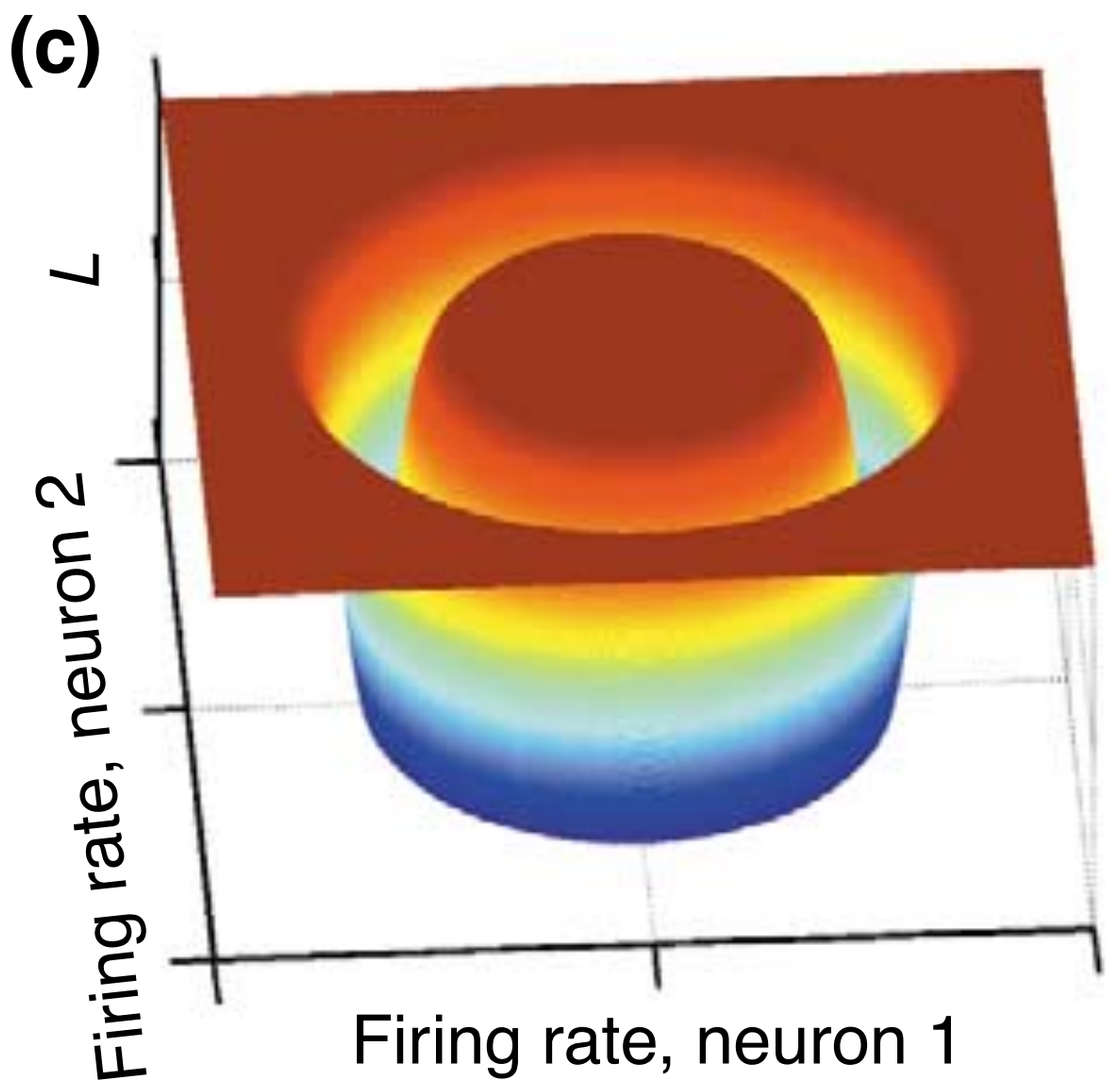
$$\frac{dI_i}{dt} = -\frac{I_i}{\tau_s} + \sum_j w_{ij} \sum_k \delta(t - t_k^j)$$


RNN
inputs
output
recurrent
units

$$\frac{d\mathbf{v}}{dt} = -\nabla E$$

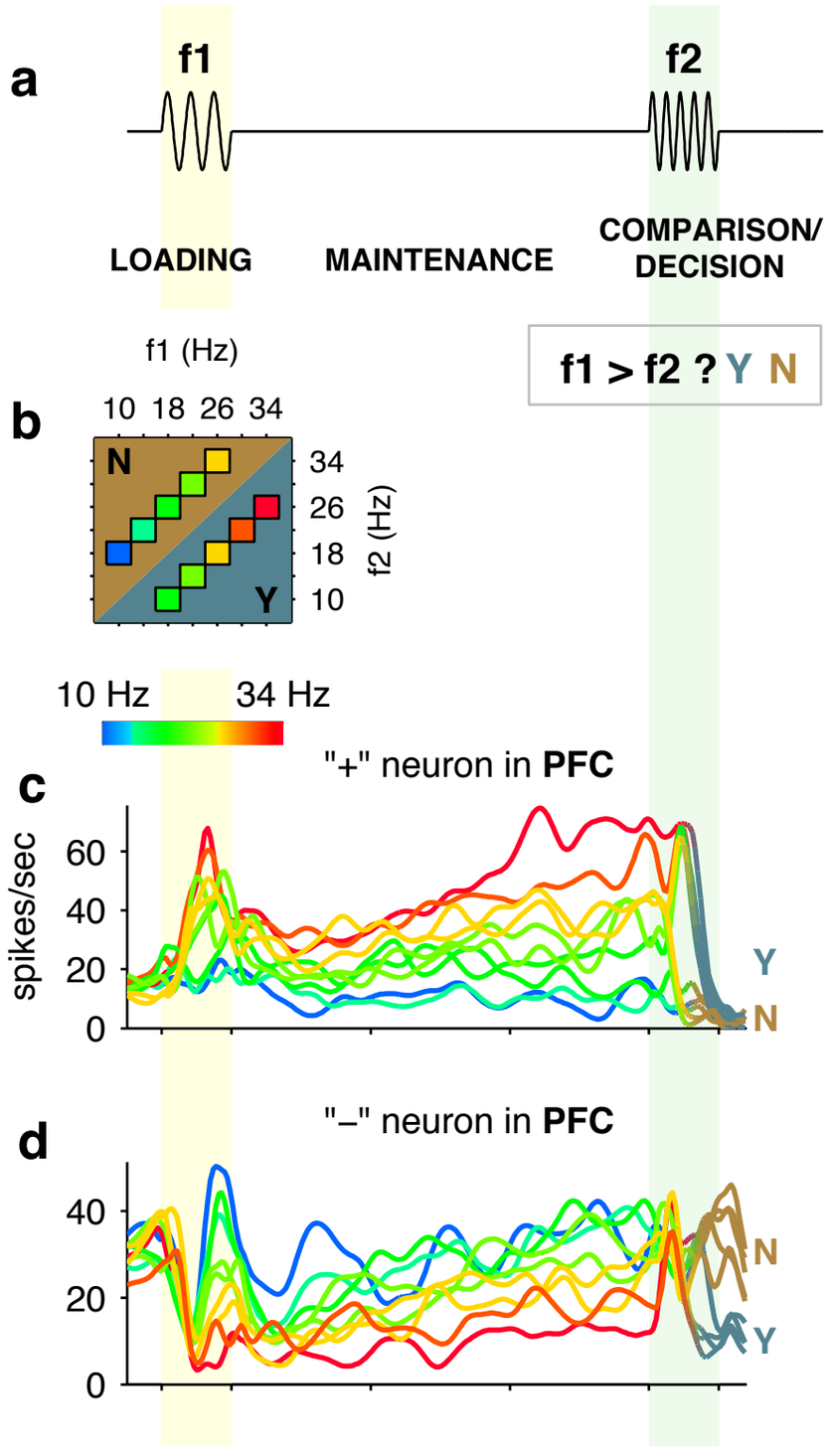


Energy landscape of a continuous attractor



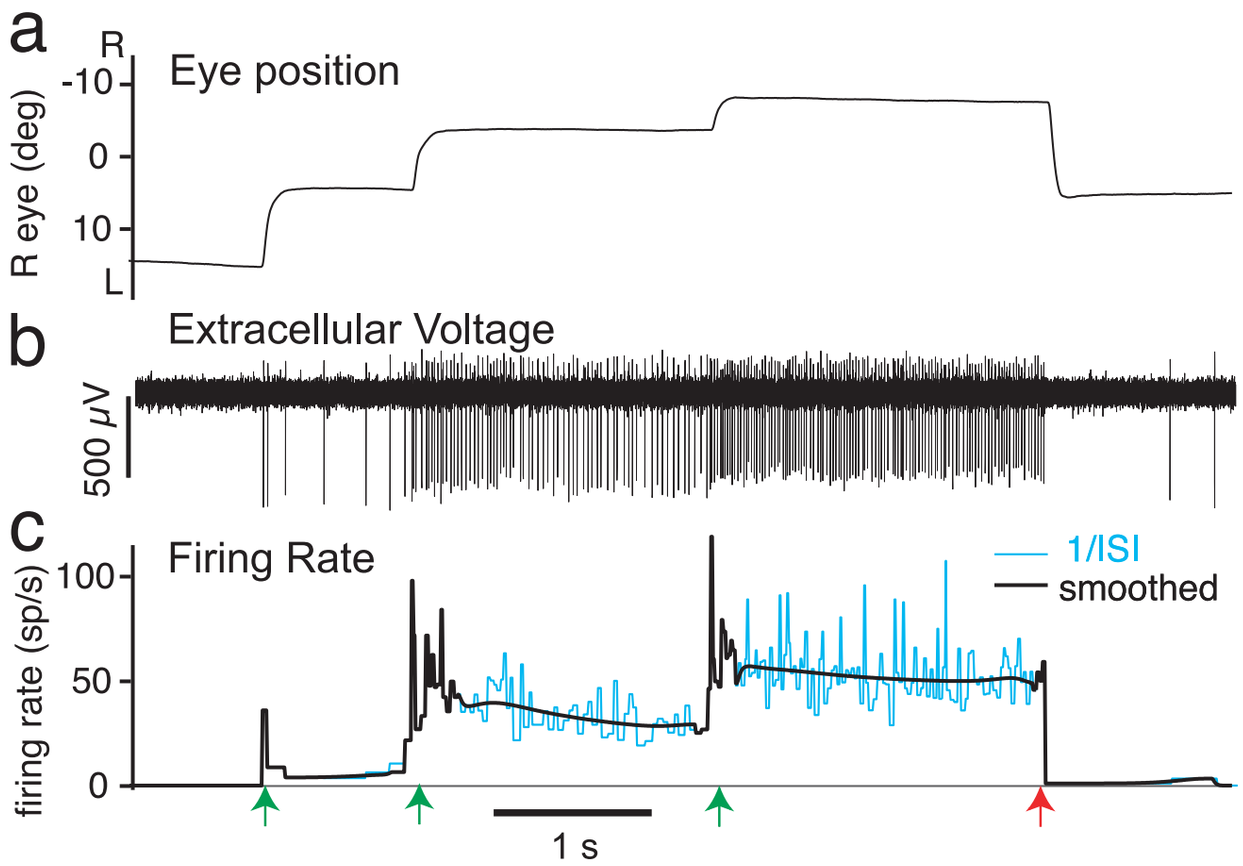
Kepecs and Brody, Current Opinion in Neurobiology 2003

A parametric working memory task

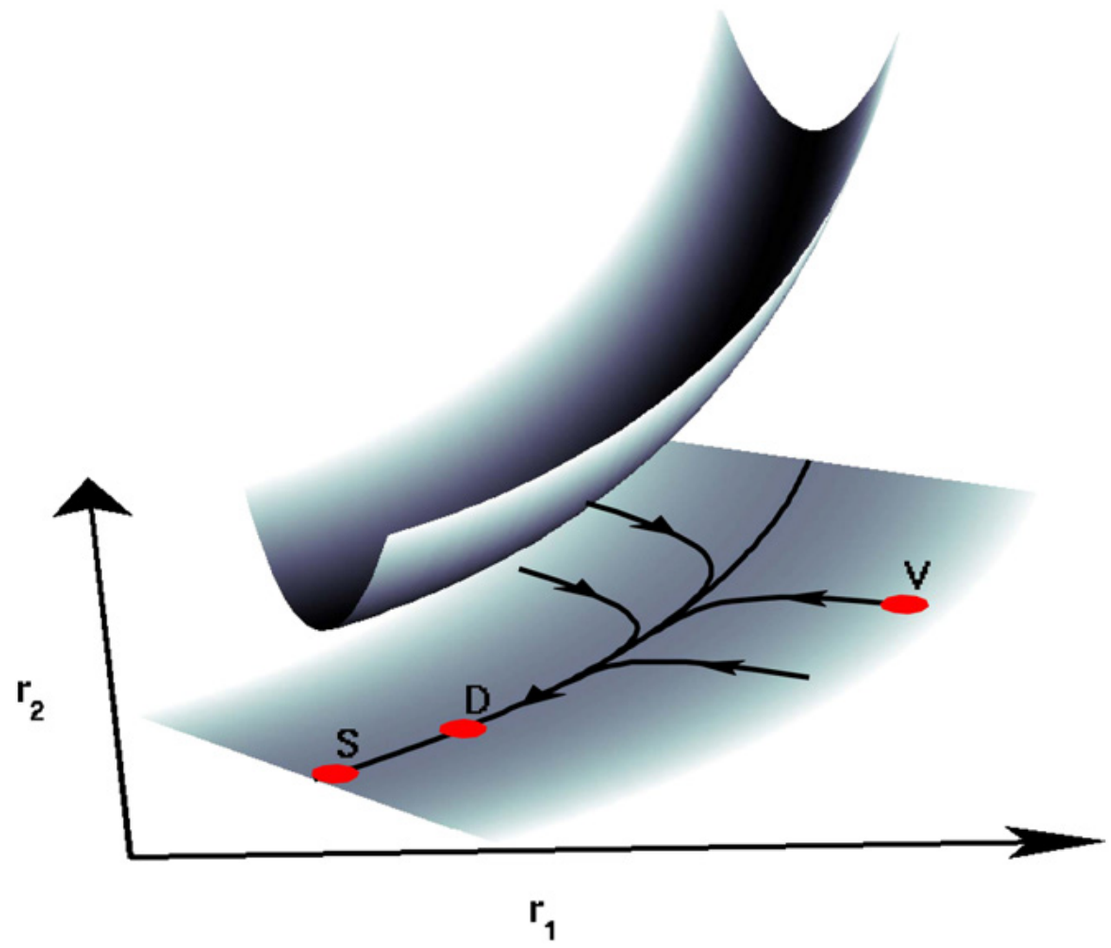


Romo et al 1999

Persistent activity in neurons encoding eye position

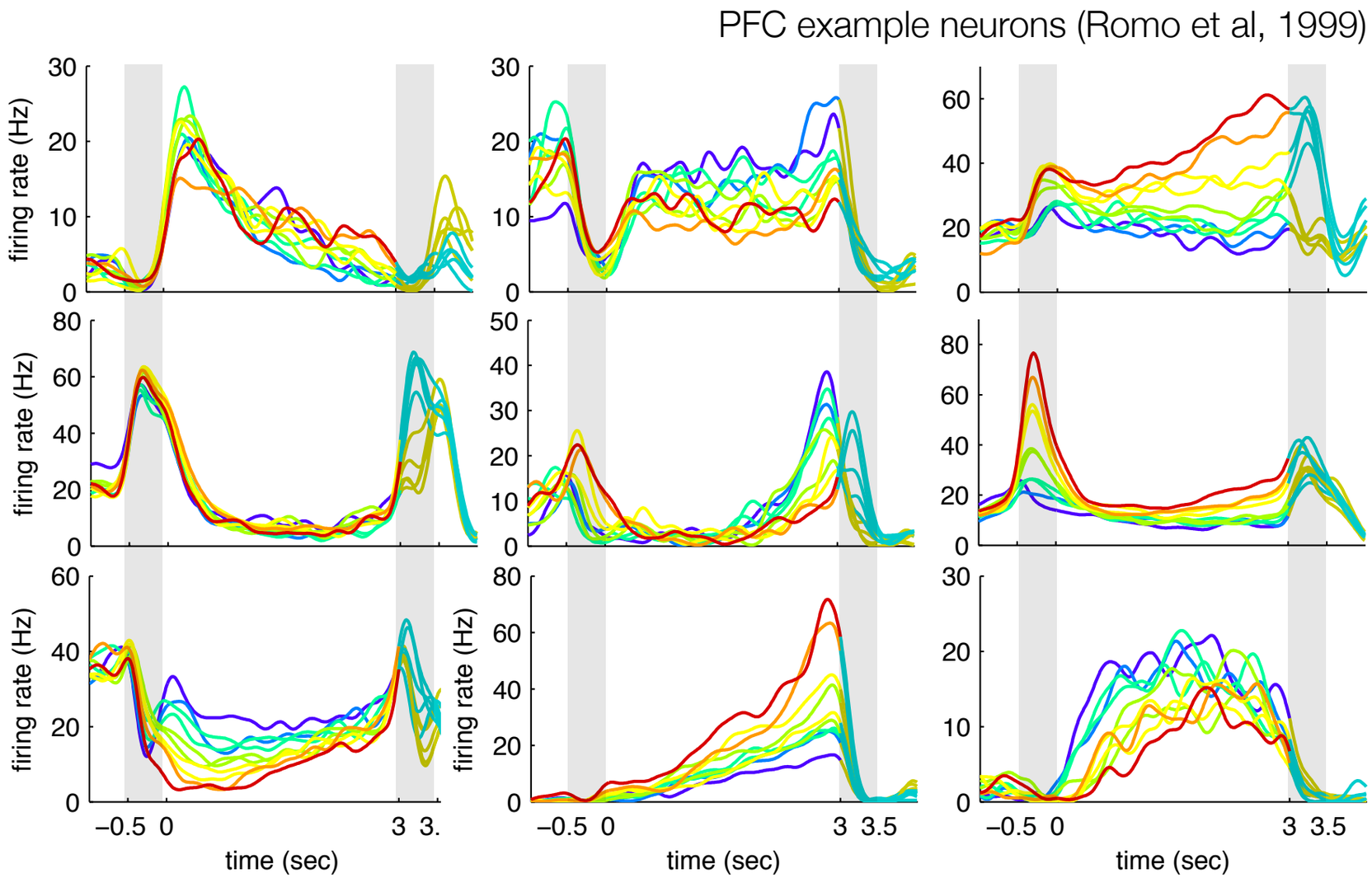


One dimensional dynamics

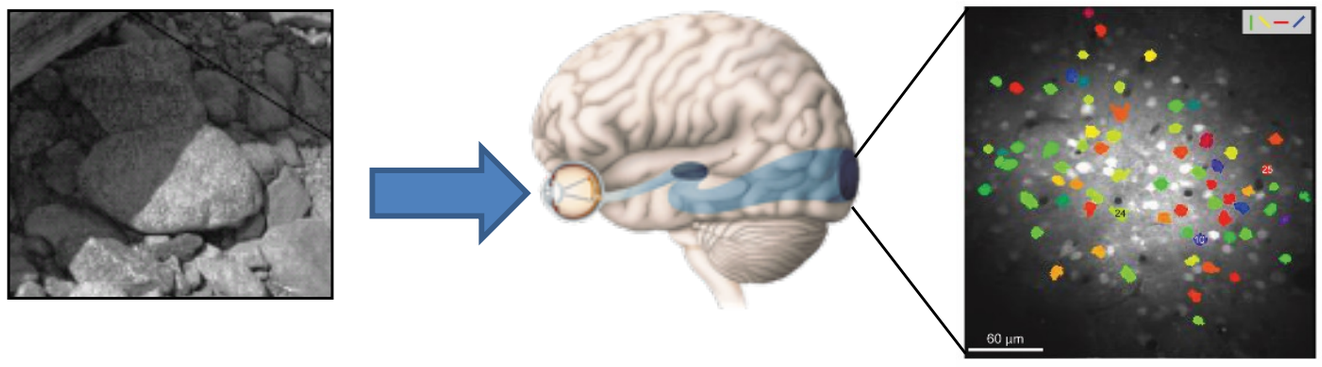


Mixed Selectivity

PFC, working memory task
PSTHs of nine (out of~1000) cells



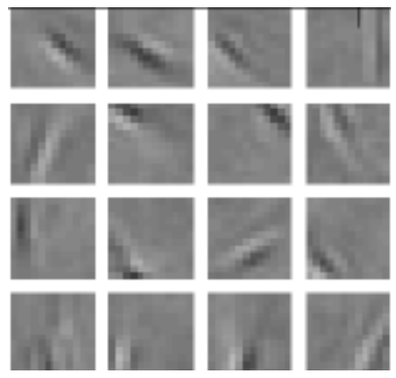
Working memory and over-representation



$$\mathbf{s} \approx \sum_{i=1}^{N} r_i(t)\mathbf{d_i}$$

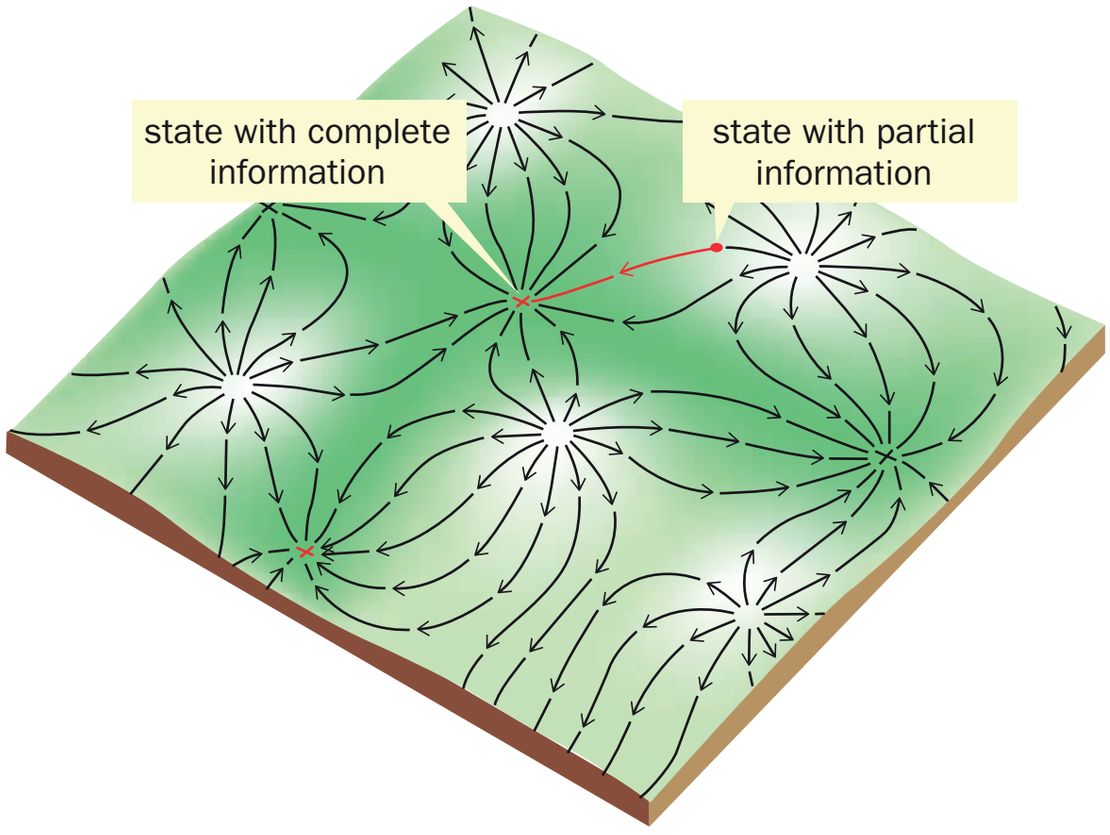
$$\frac{d\mathbf{s}}{dt} = 0$$

$\mathbf{d}_{1:N}$



Shaul Drukmann et al 2013

Memory recall and Hopfield network



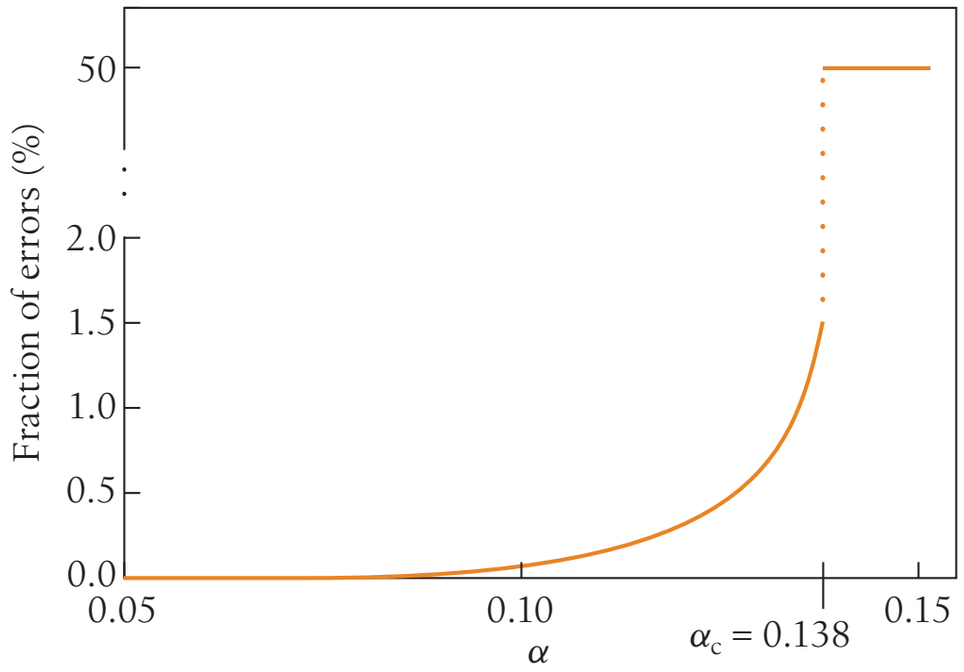


FIGURE 5.51

Recall of memories in the Hopfield model. The number of patterns K stored in the network is proportional to the number of neurons, so that $\alpha = K/N$ is the natural parameter. As α increases, the fraction of errors in the recalled patterns increases, until at some critical value α_c recall fails catastrophically, and the states of the network are not correlated with the stored patterns at all. Redrawn from Amit et al. (1985).

The Basic Architecture of Transformer

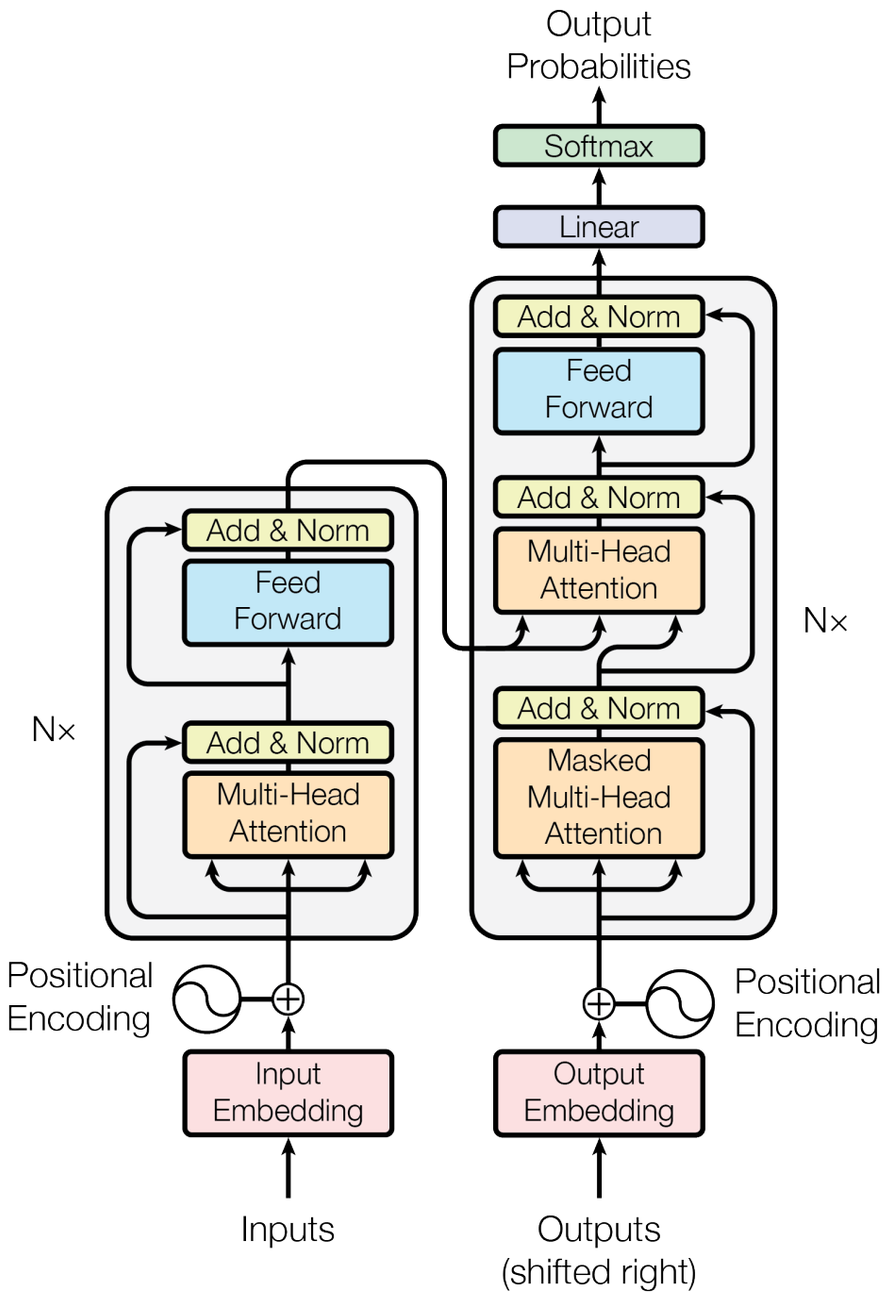
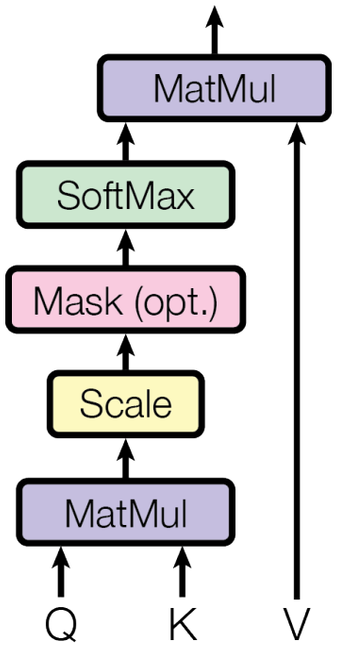


Figure 1: The Transformer - model architecture.

Scaled Dot-Product Attention



$$\text{Attention}(Q, K, V) = \text{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$

Attention is all you need

Fixed points and stability

Global stability: starting from **any** initial condition

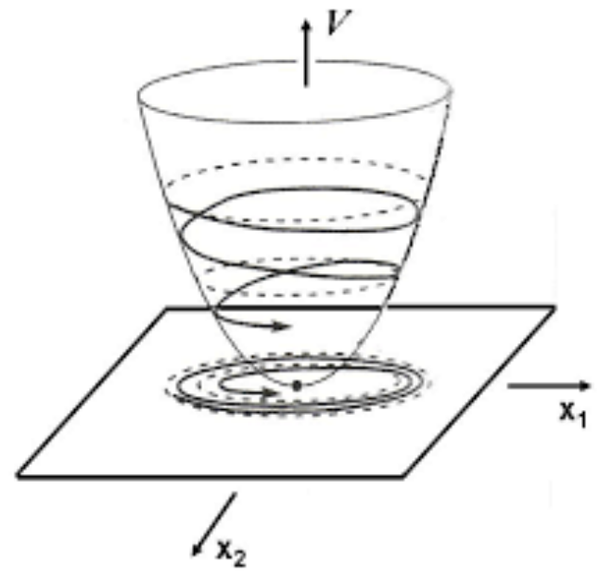
$$\frac{d\mathbf{u}}{dt} = \mathbf{f}(\mathbf{u})$$

Lyapunov function $V(u)$

$V(u) \geq \mathrm{V}_0$, has a lower bound

$$\frac{d}{dt}V(u(t)) \leq 0$$

$$\frac{d}{dt}V\big(u(t)\big) = 0 \Rightarrow \nabla V(u) = 0$$



Theorem: If there exists a Lyapunov function, the system is (globally) stable where the trajectory will converge to one of the extrema of $V(u)$.

An example

$$\frac{dx}{dt} = -x + 4y$$

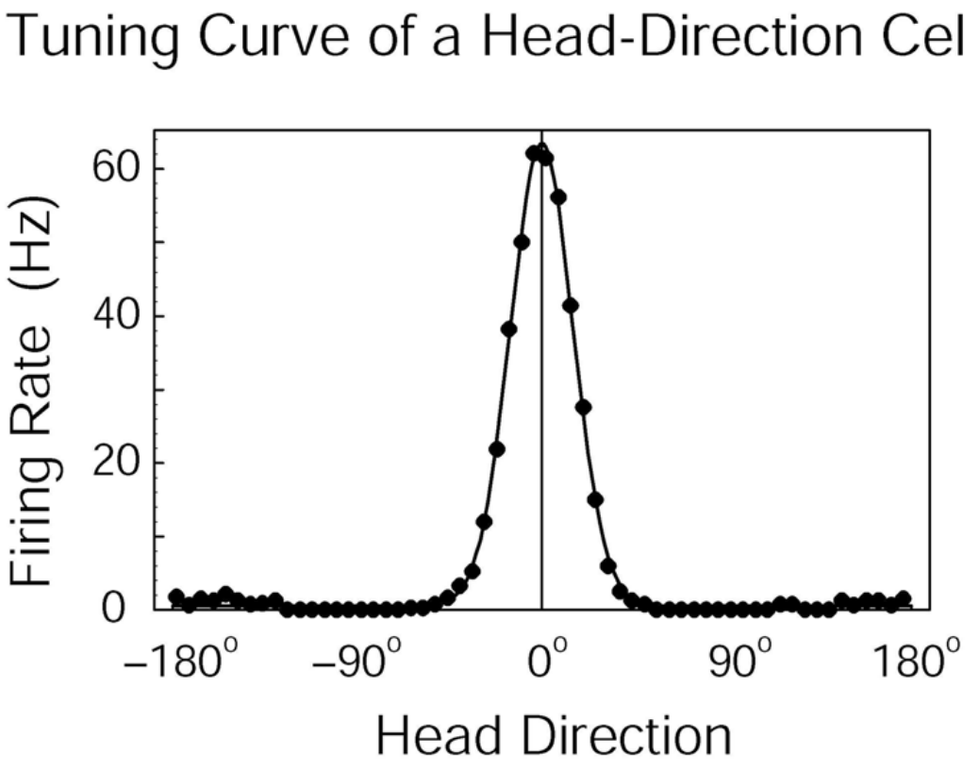
$$\frac{dy}{dt} = -x - y^3$$

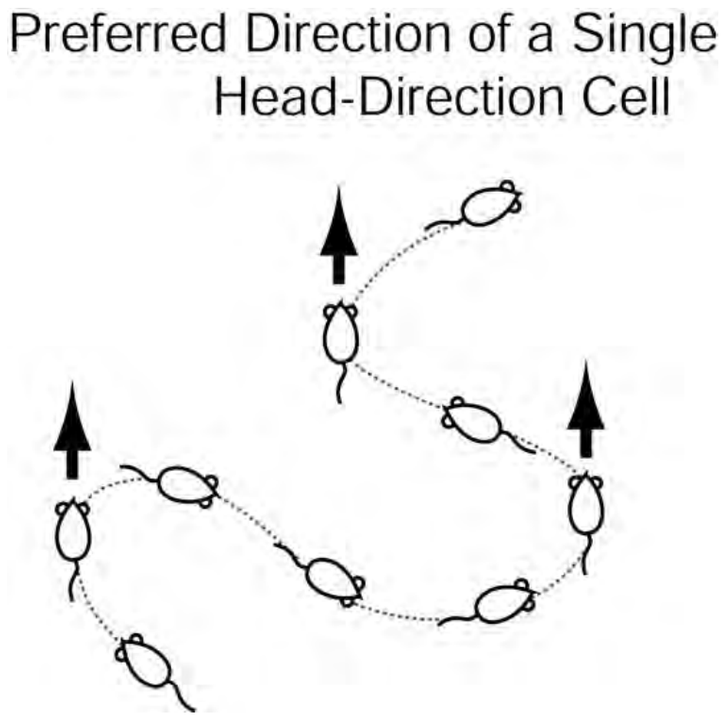
Consider $V(x, y) = x^2 + \alpha y^2$

$$\dot{V} = 2x\dot{x} + 2\alpha y\dot{y} = 2x(-x + 4y) + 2\alpha y(-x - y^3)$$

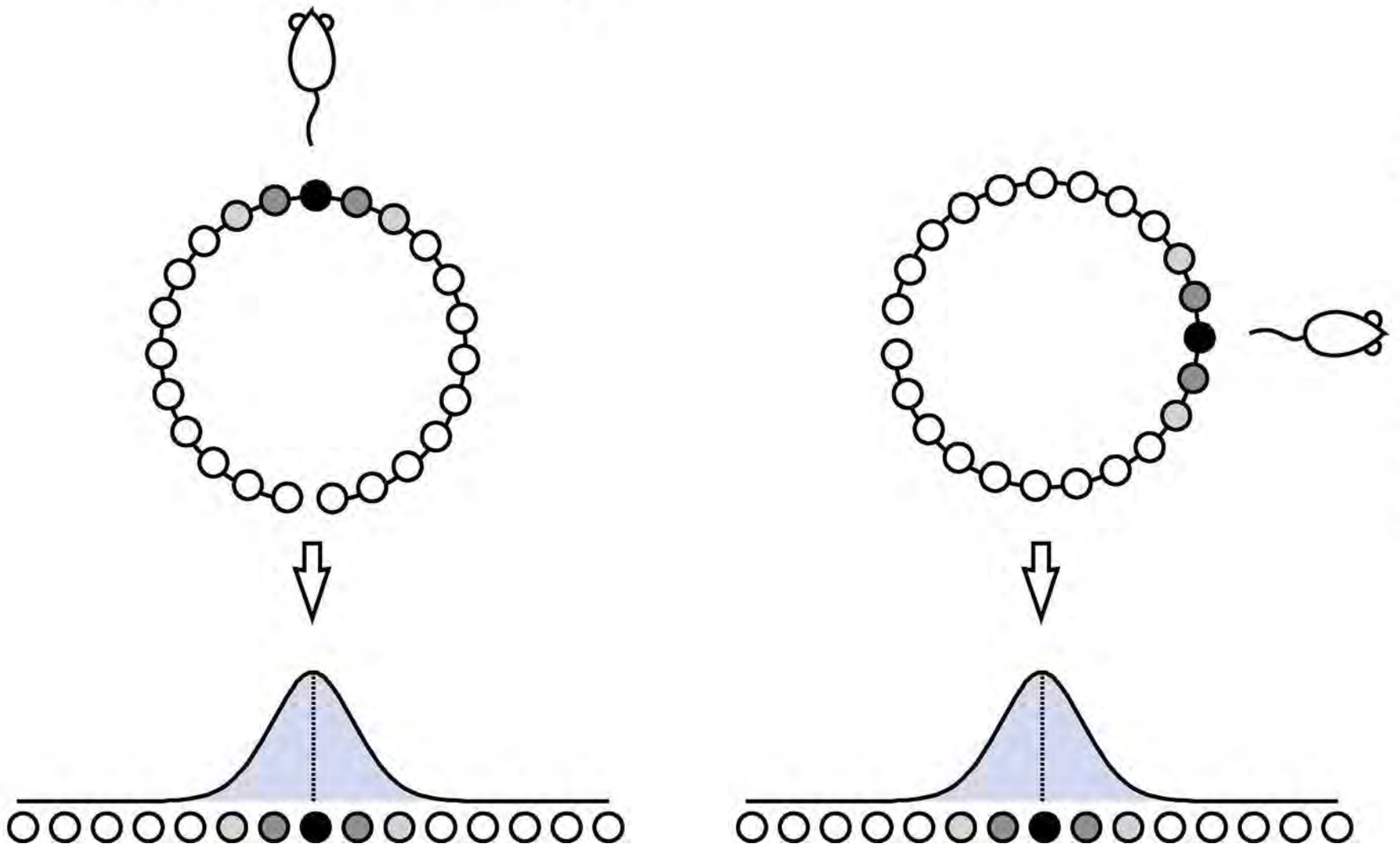
$$= -2x^2 - 2\alpha y^4 + (8 - 2\alpha)xy$$

Head direction cell





A ring attractor model

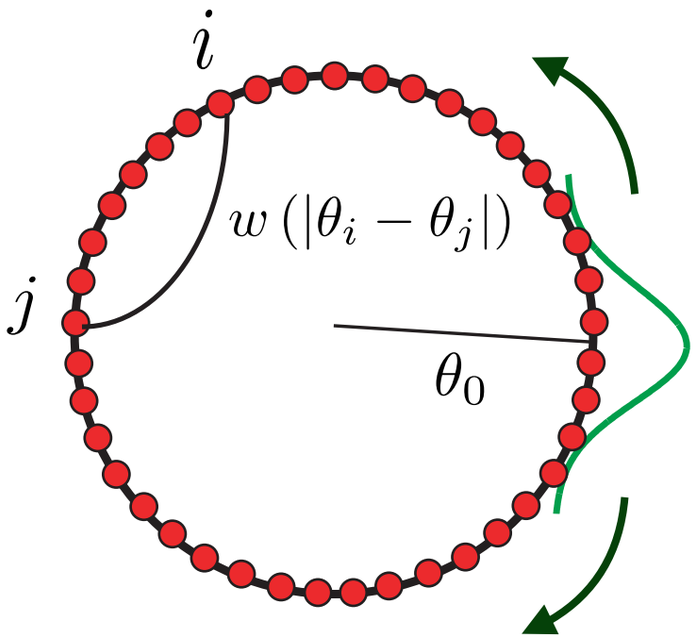


Ben-Yishai, Bar-Or, Sompolinsky, PNAS (1995)

Zhang, J. Neurosci. (1998)

Xie, Hahnloser, Seung, Phys. Rev. E (2002)

A ring attractor model



firing rate

$u_i = f(\theta_i - \theta_0)$

Solution:

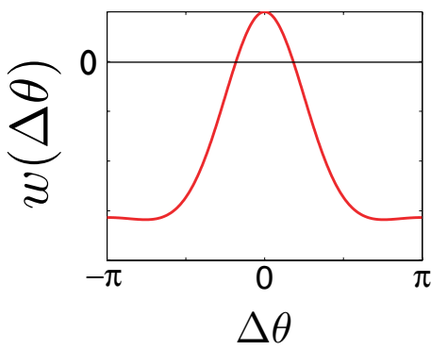
A continuum of steady states
encode a continuous parameter (angle)

e.g., $u(\theta) = a + b\cos(\theta - \theta_0)$

local excitation, long range inhibition

$$\tau \frac{du_i}{dt} = -u_i + \left[\sum_{j=1..N} w_{ij}u_j + I_i^0\right]_+$$

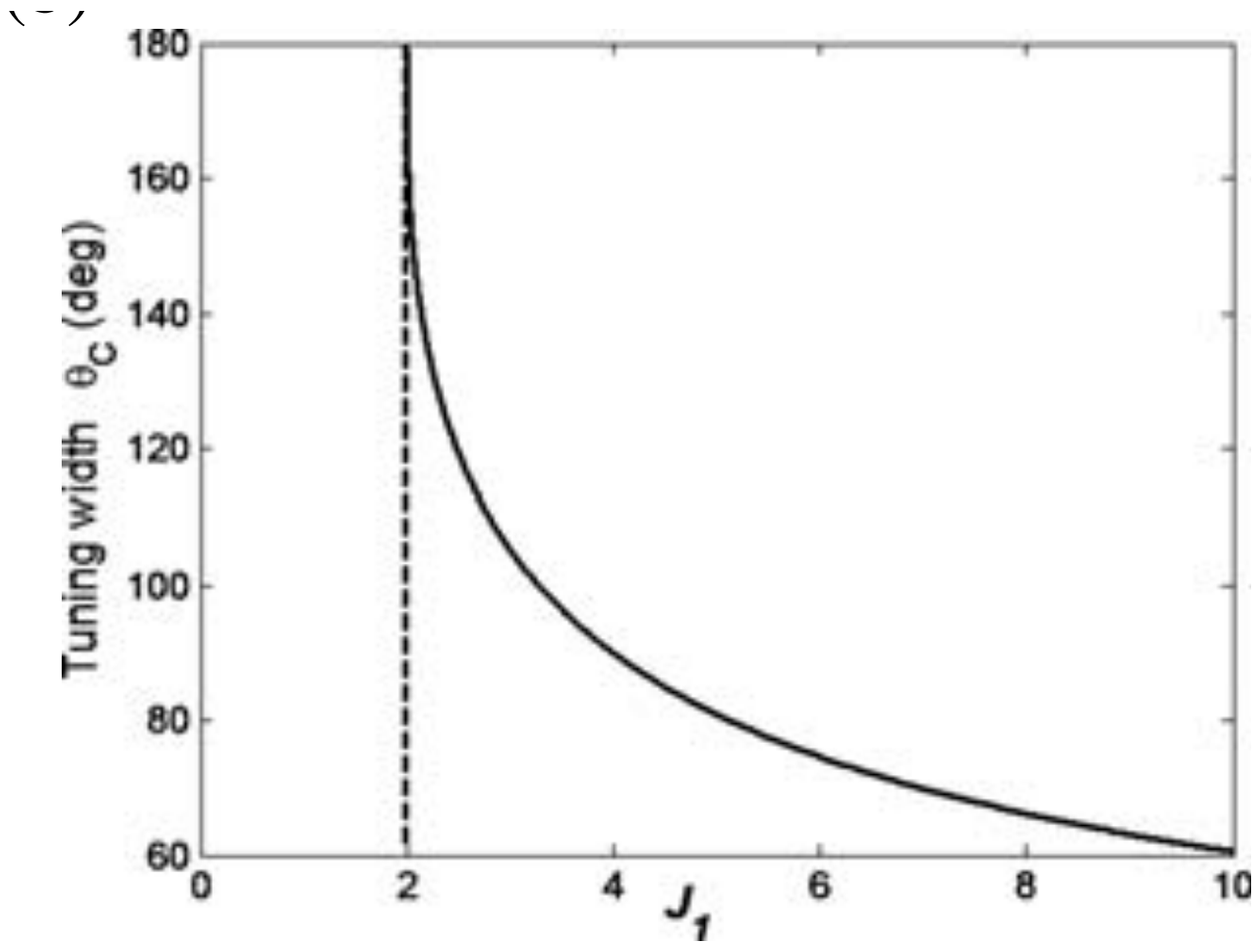
$$w_{ij} = w_{ji} = \frac{1}{N} J(|\theta_i - \theta_j|)$$



$$J = J_0 + J_1 \cos\theta$$

Let $N \to \infty$

$$\tau \frac{\partial u(\theta, t)}{\partial t} = -u(\theta, t) + \left[\frac{1}{2\pi}\int_{-\pi}^{\pi} J(\theta - \theta')u(\theta', t)d\theta' + I^0(\theta)\right]_+$$


180
160
140
120
100
80
60
0
2
4
6
8
10
J_1
Tuning width θ_C (deg)

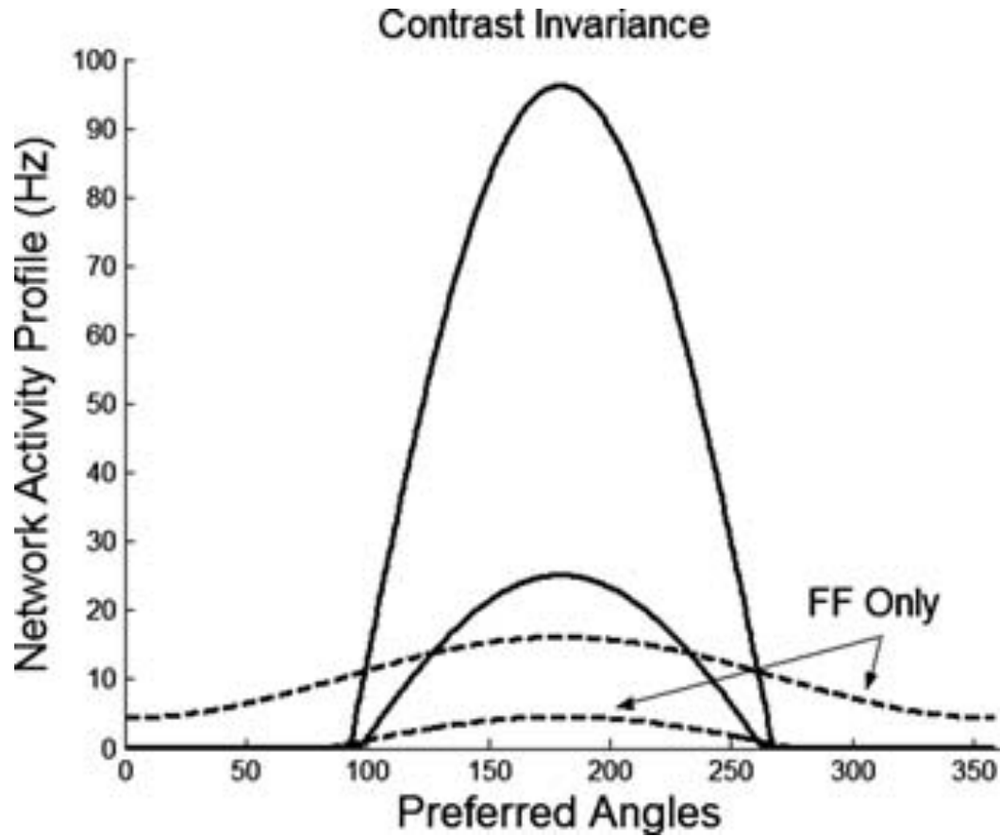
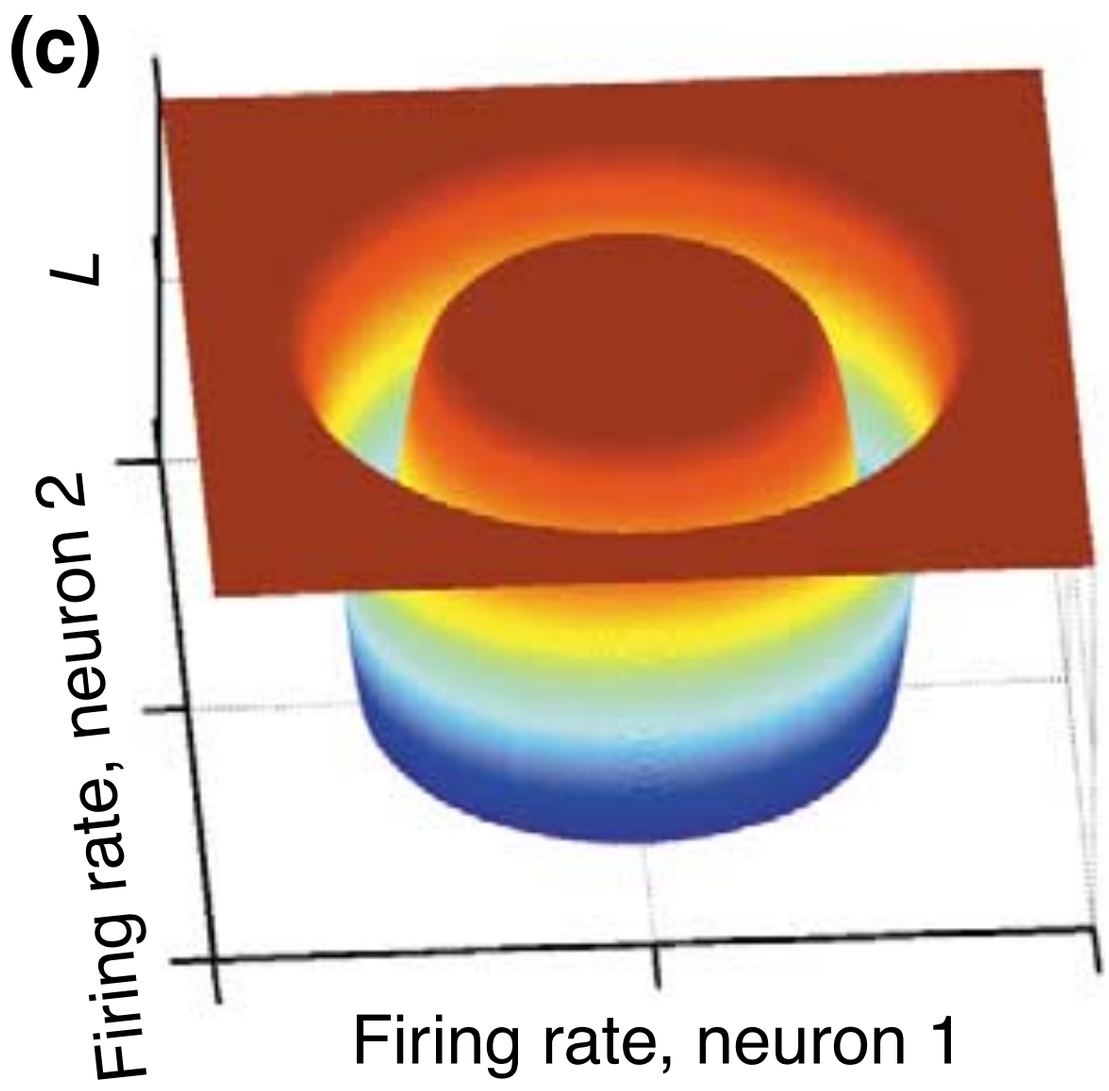


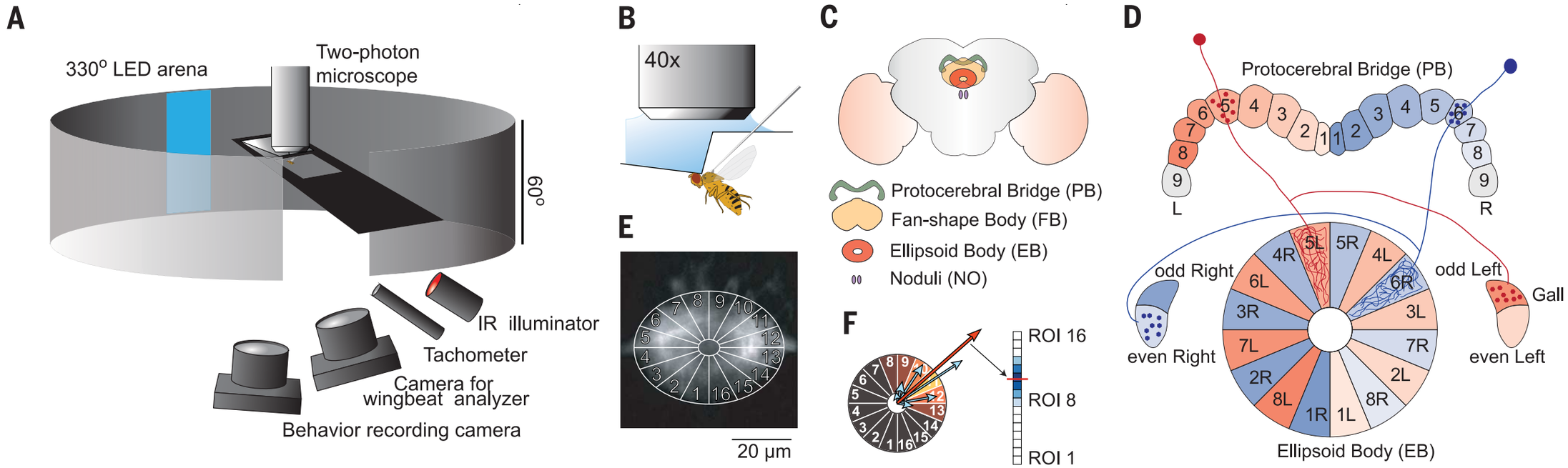
Fig. 5.2. Activity profile of the ring network for different contrasts.

Energy landscape of a continuous attractor

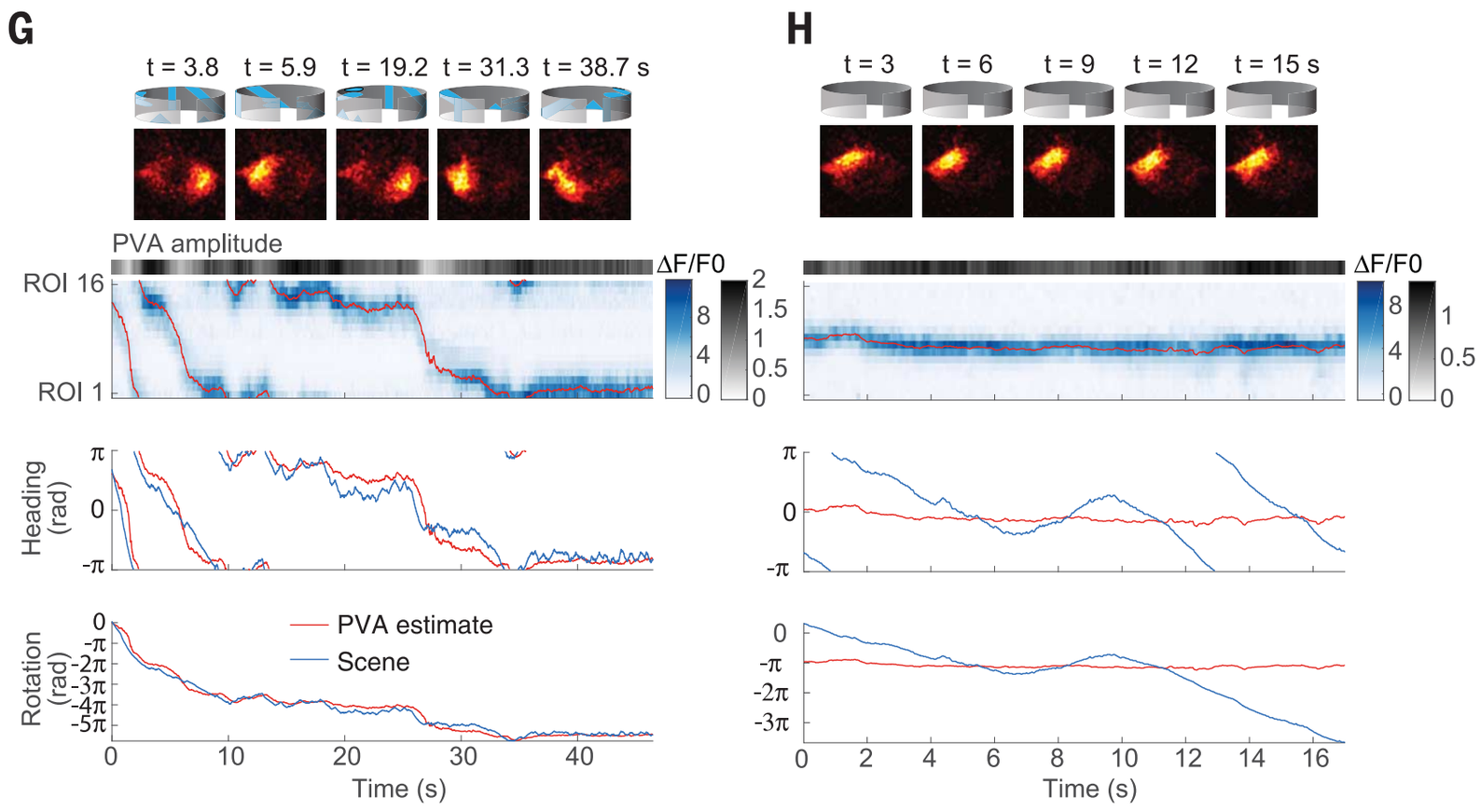


Kepecs and Brody, Current Opinion in Neurobiology 2003

Ring attractor dynamics in fly brain



Ring attractor dynamics in fly brain



Ring attractor dynamics in fly brain

