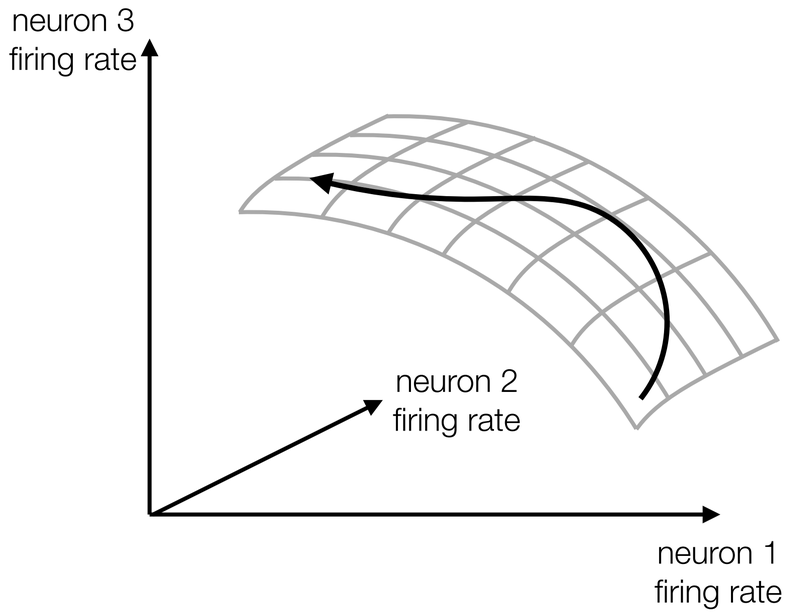
Demixed neural activity

Variance is not importance.

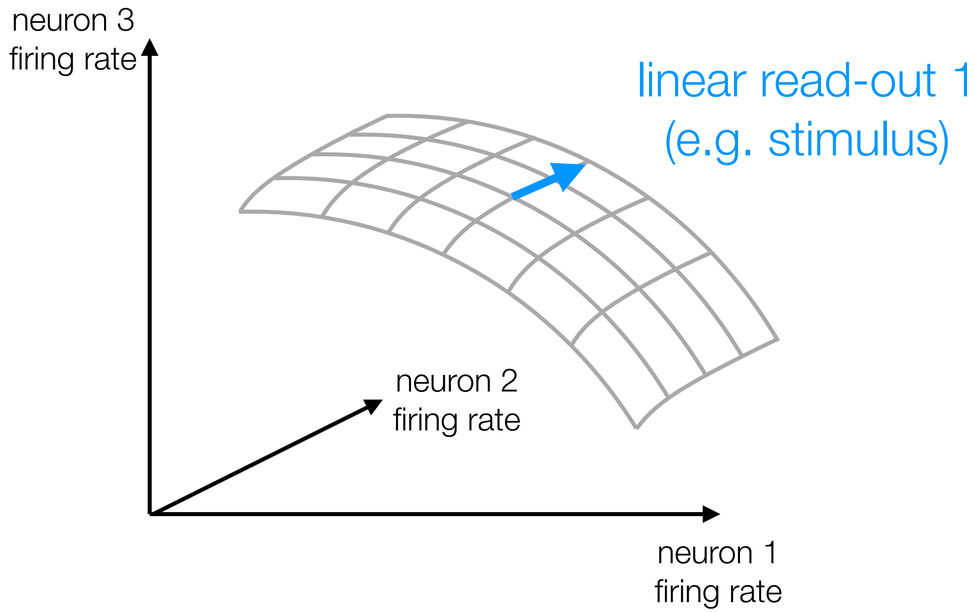
A dimension can carry little variance and great behavioral relevance.

PCA optimizes a criterion nobody claimed the brain uses.

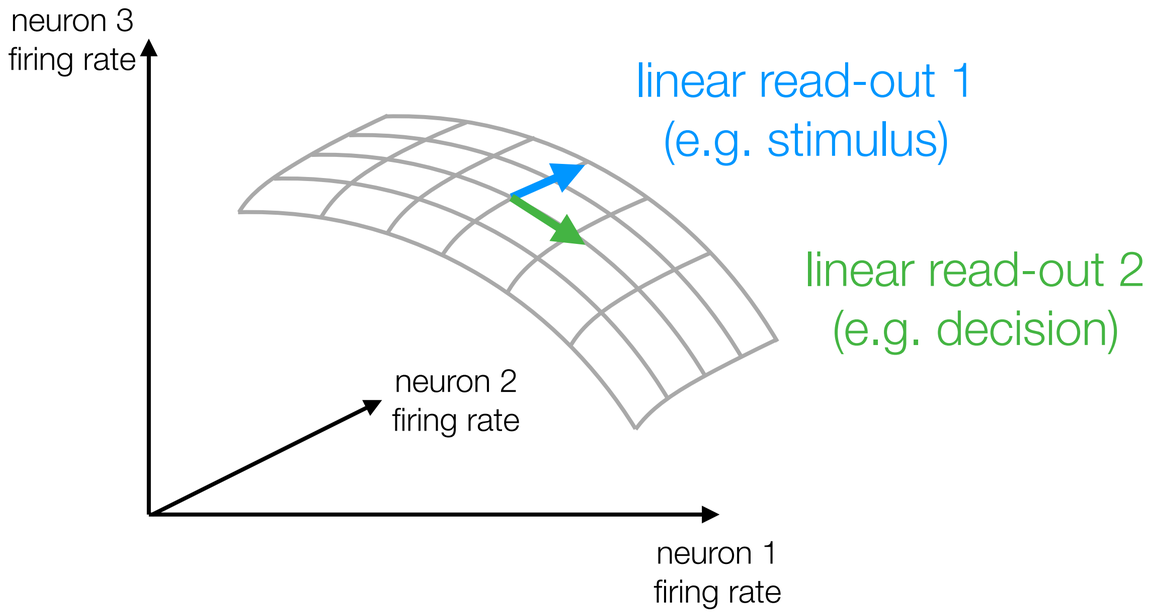
Can we assign meaning to the neural manifold?

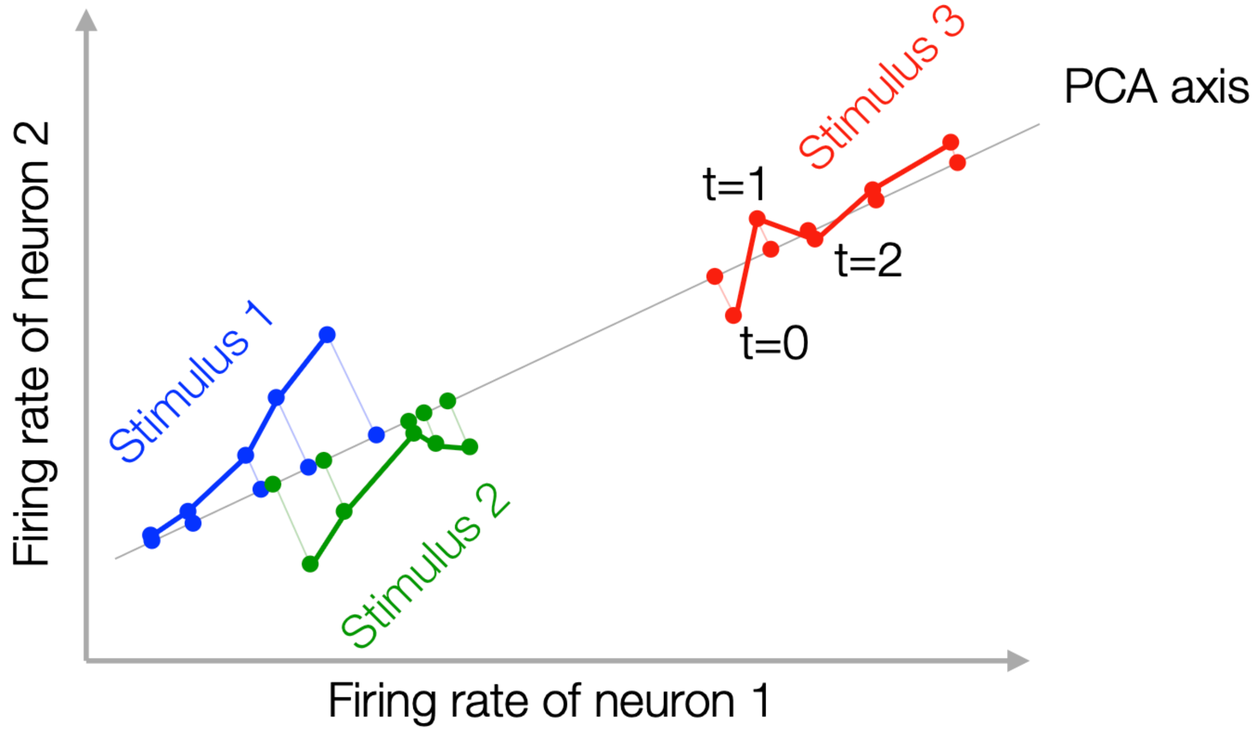


Can we assign meaning to the neural manifold?

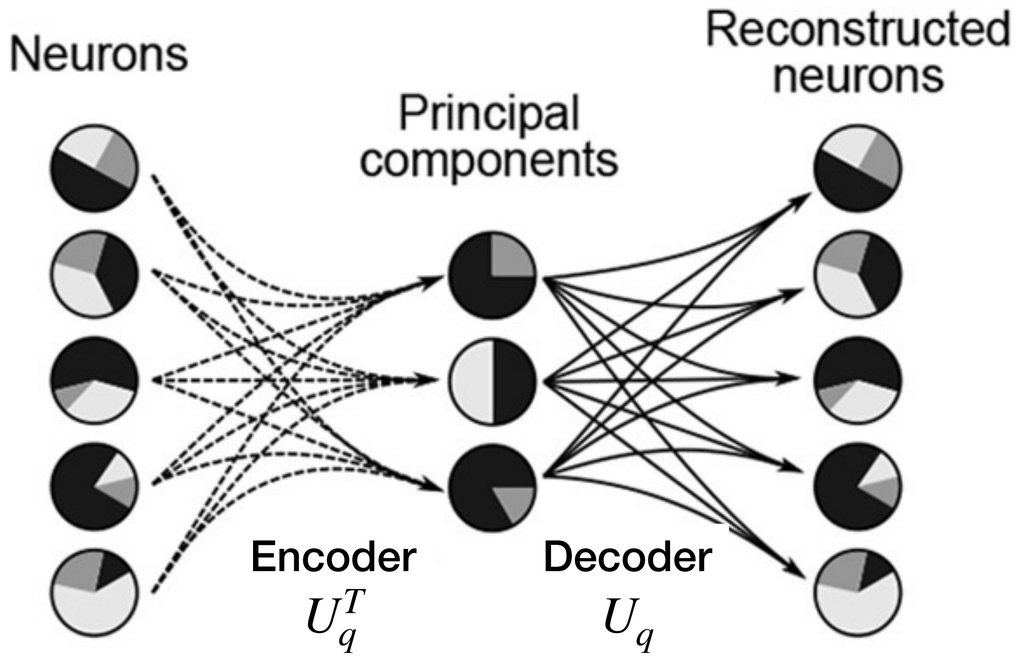


Can we assign meaning to the neural manifold?

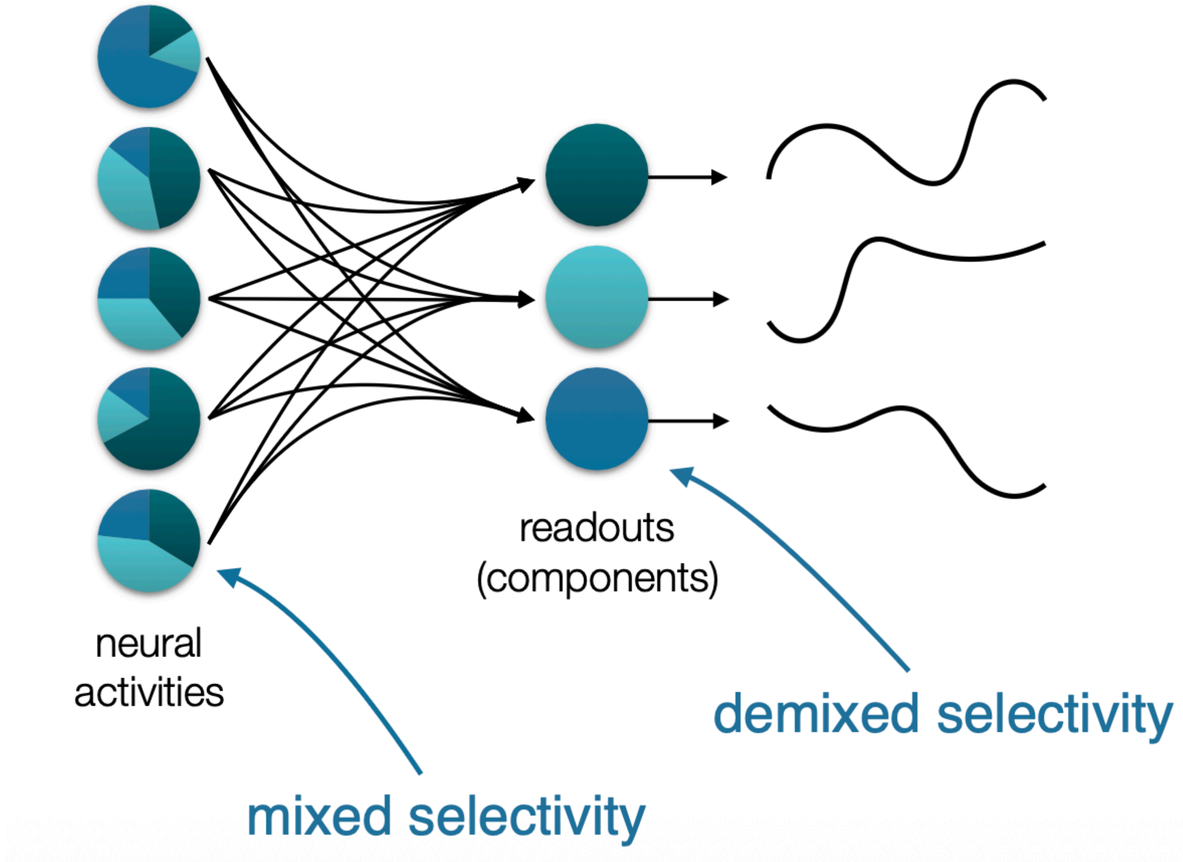



Firing rate of neuron 2
Firing rate of neuron 1
PCA axis
Stimulus 1
Stimulus 2
Stimulus 3
t=0
t=1
t=2

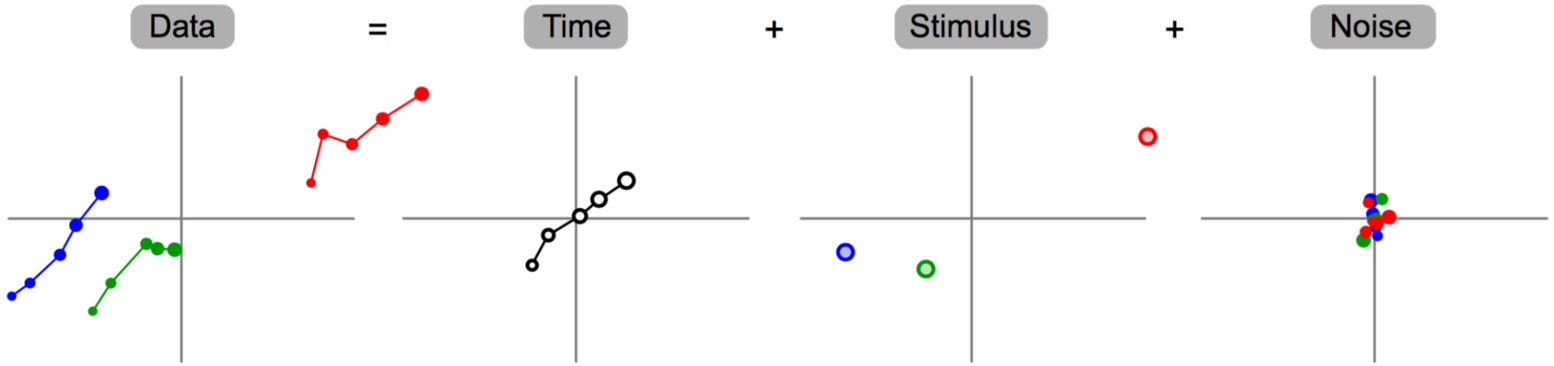
PCA


Neurons
Principal components
Reconstructed neurons
Encoder
U_q^T
Decoder
U_q

Demixed PCA


readouts
(components)
neural
activities
demixed selectivity
mixed selectivity

Decompose data into marginalized averages

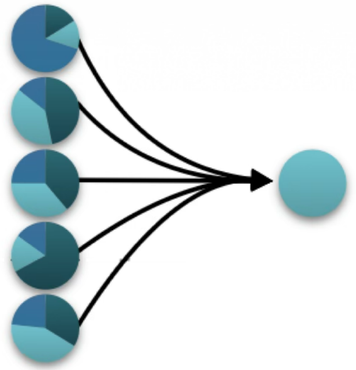


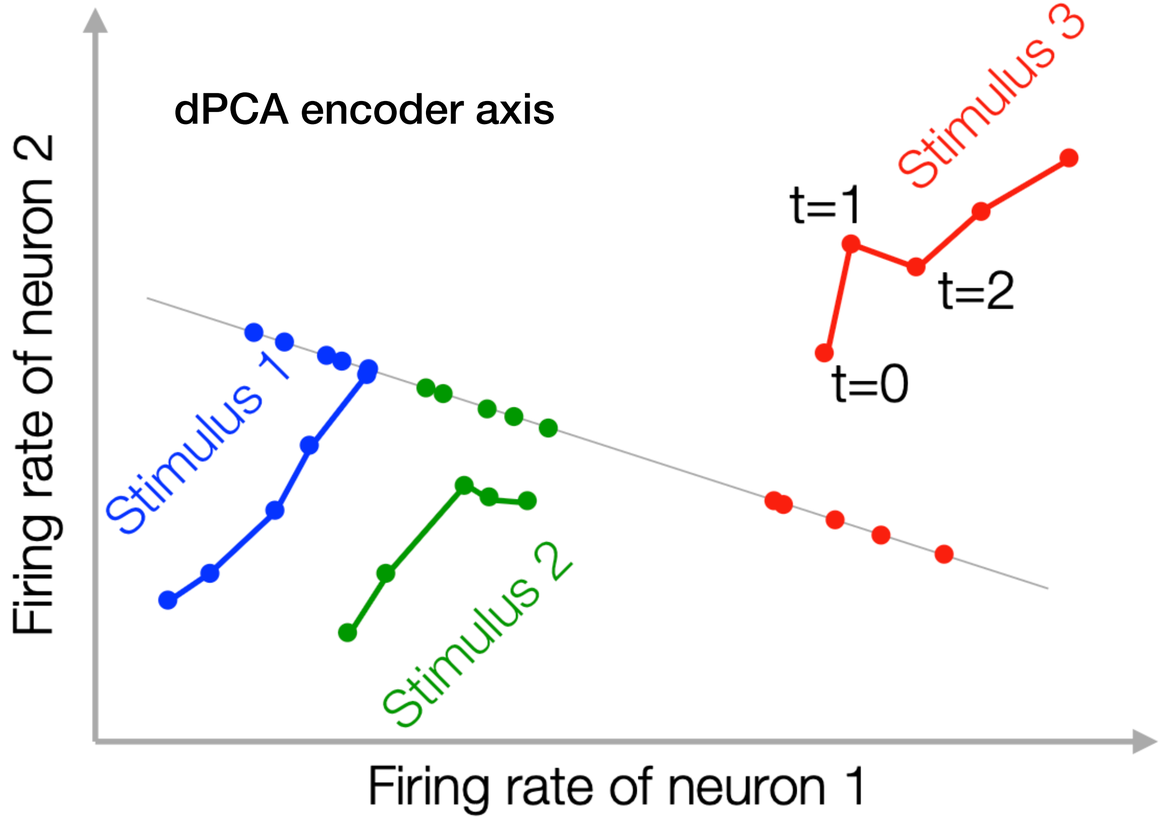
$$\mathbf{X} = \mathbf{X}_t + \mathbf{X}_s + \mathbf{X}_{st}$$

$$X_t = \langle X \rangle_s$$

$$X_s = \langle X \rangle_t$$

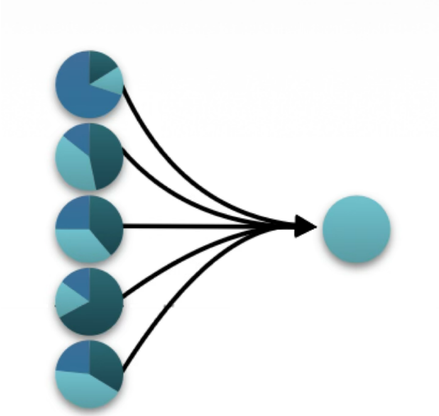
Linear readout with dPCA

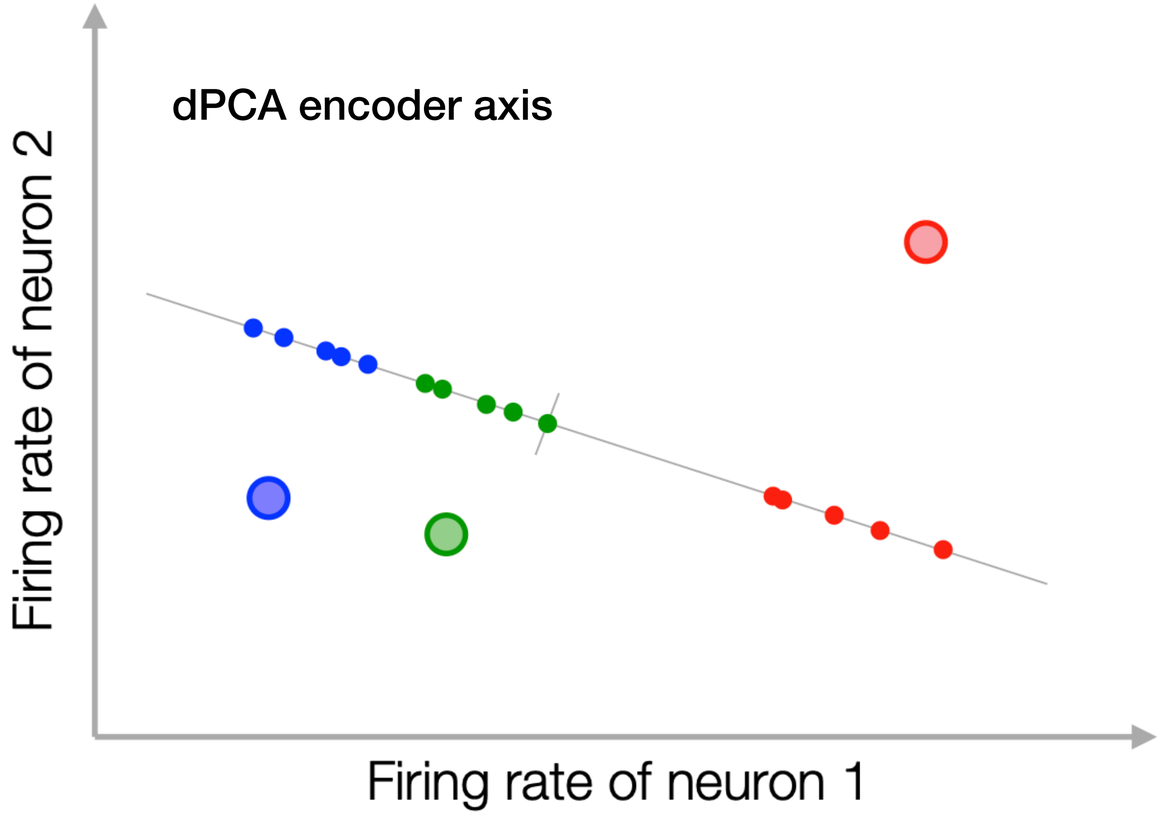




Kobak et al. *eLife* 2016

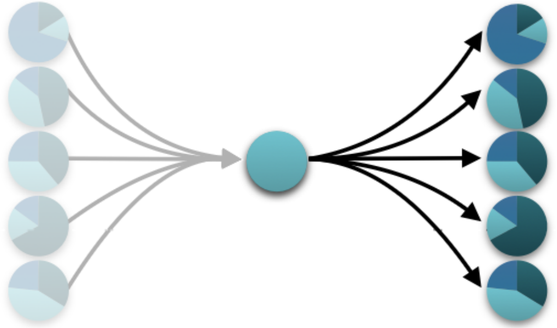
Linear readout with dPCA

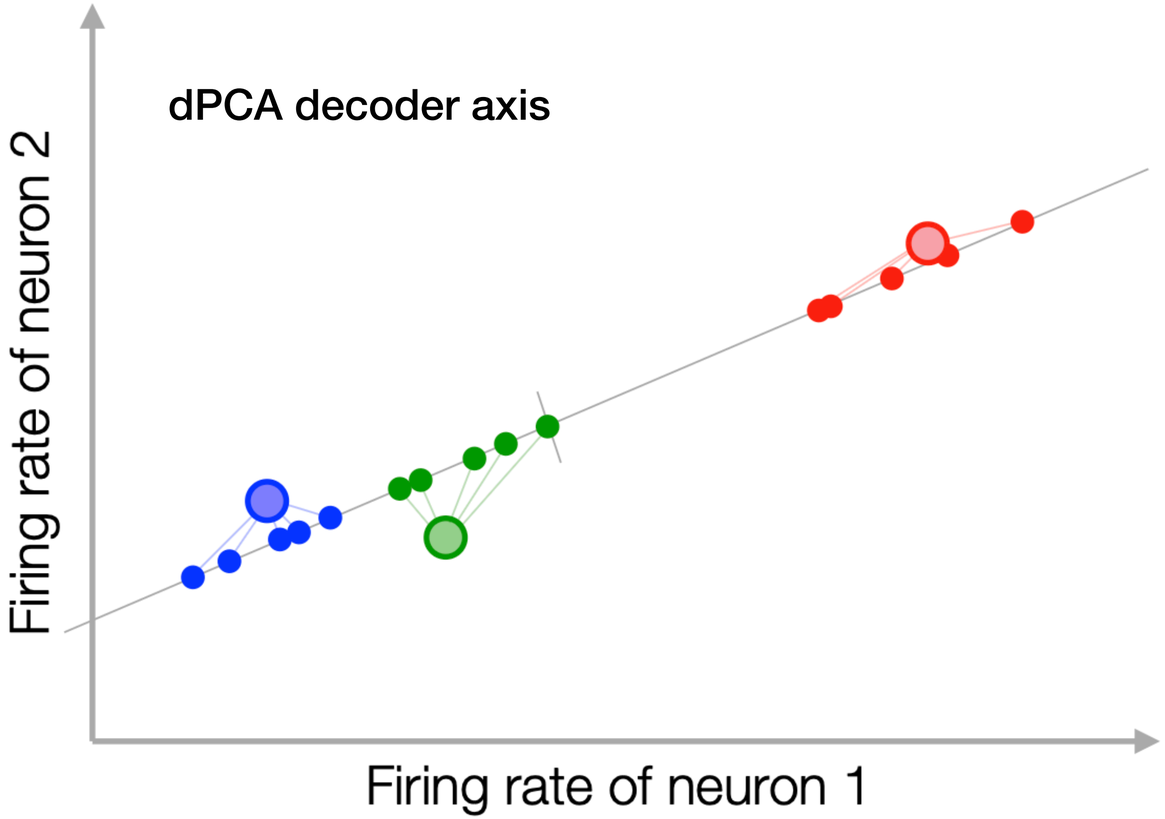




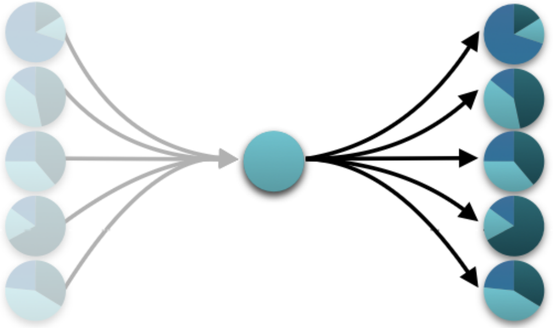
Kobak et al. *eLife* 2016

Linear readout with dPCA





Kobak et al. *eLife* 2016



Loss function:

$$L = \|\mathbf{X}_s - \mathbf{FDX}\|^2$$

‘marginalized’ averages

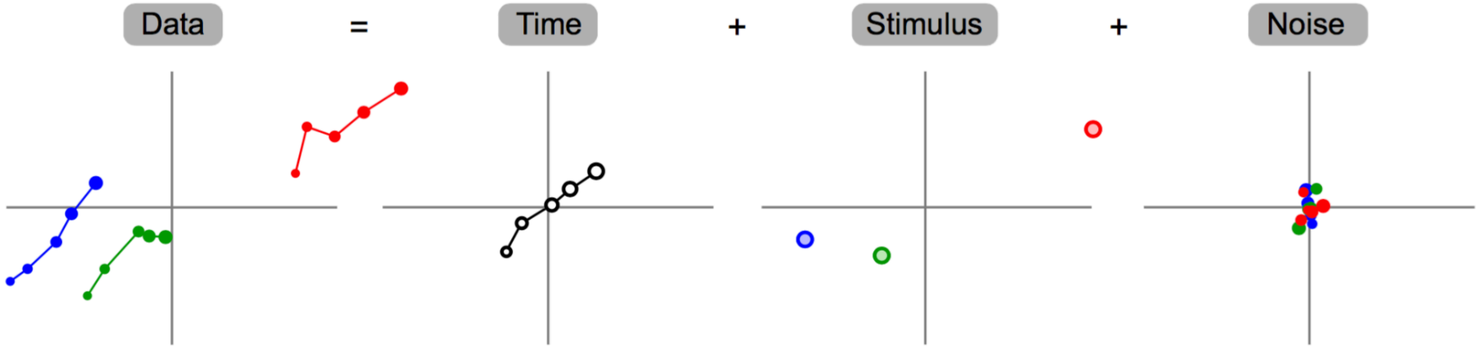
neural activities

decoder

encoder

reduced rank regression!

Separate encoders/decoders for marginalized averages



$$\mathbf{X} = \mathbf{X}_t + \mathbf{X}_s + \mathbf{X}_{st}$$

$$L = \|X_t - F_t D_t X\|^2 + \|X_s - F_s D_s X\|^2 + \|X_{ts} - F_{ts} D_{ts} X\|^2 = \sum_{\phi} \|X_\phi - F_\phi D_\phi X\|^2$$

One rank-constrained regression per marginalization: source X, target X_ϕ, rank q.

dPCA is reduced-rank regression, term by term

$$\|X_\phi - F_\phi D_\phi X\|^2 = \|X_\phi - \hat{X}_\phi\|^2 + \|\hat{X}_\phi - B_q X\|^2, \quad B_q = F_\phi D_\phi$$

$$\hat{X}_\phi = X_\phi P = X_\phi X^\top (XX^\top)^{-1} X = B_{OLS} X$$

$$\min_{B_q} \|B_{OLS} X - B_q X\|^2$$

Fit with least square regression, then find the optimal q-rank truncation.

dPCA is reduced-rank regression, term by term

$$\|X_\phi - F_\phi D_\phi X\|^2 = \|X_\phi - \hat{X}_\phi\|^2 + \|\hat{X}_\phi - B_q X\|^2, \quad B_q = F_\phi D_\phi$$

$$\hat{X}_\phi = X_\phi P = X_\phi X^\top (XX^\top)^{-1} X = B_{OLS} X$$

$$\min_{B_q} \|\hat{X}_\phi - B_q X\|^2$$

$$\hat{X}_\phi = U \Sigma V^\top$$

$$B_q X = U_q U_q^\top B_{OLS} X$$ Apply Eckart-Young theorem!

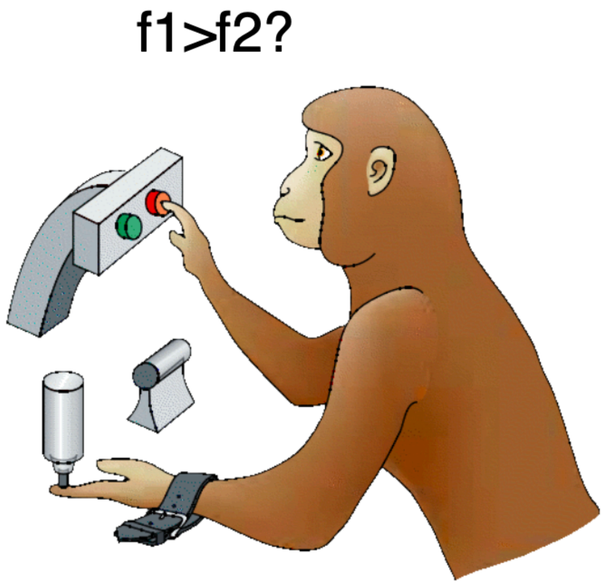
$$F_\phi = U_q \, , \, D_\phi = U_q^\top B_{OLS}$$

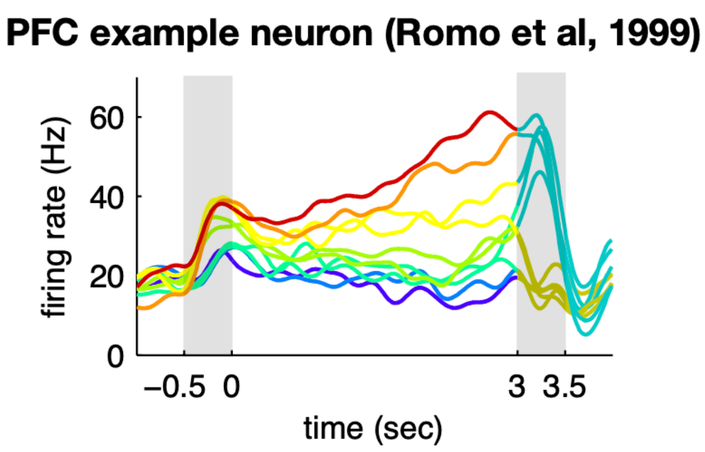
Two projections again, one on each side

$$U_q U_q^\top \; X_\phi \; P$$

- The right projector does the regression: keep only what the source can explain.
- The left projector does the dimensionality reduction: keep the top q directions of what is left.

A PFC working memory task





task parameters

stimulus f1

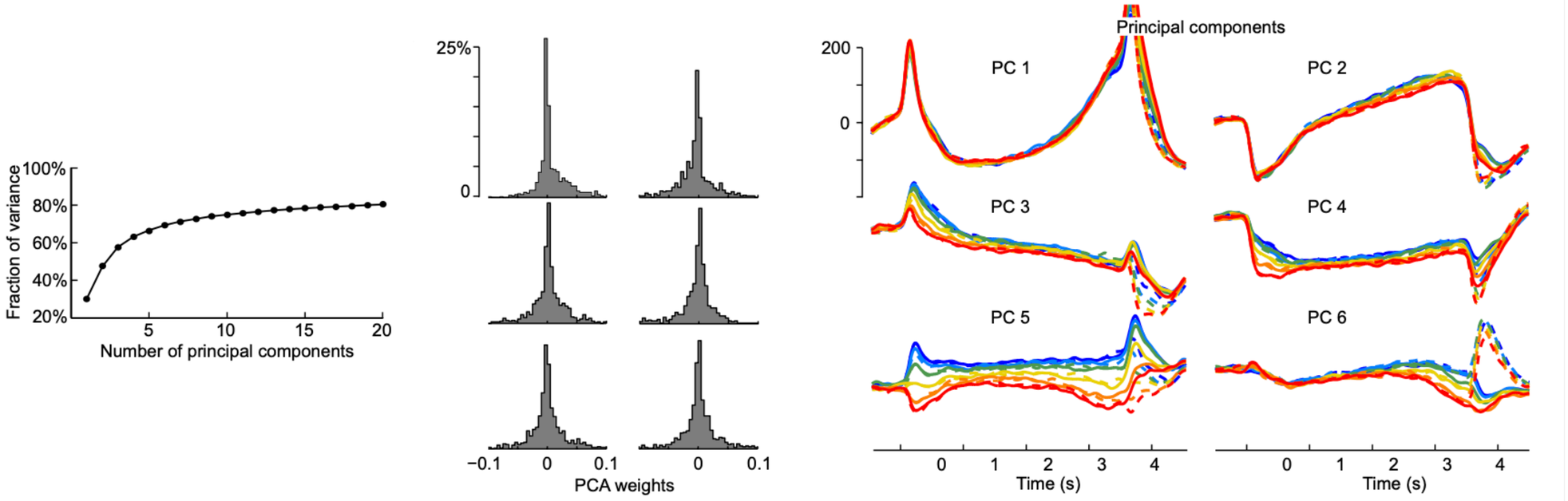
10 Hz 34 Hz

decision

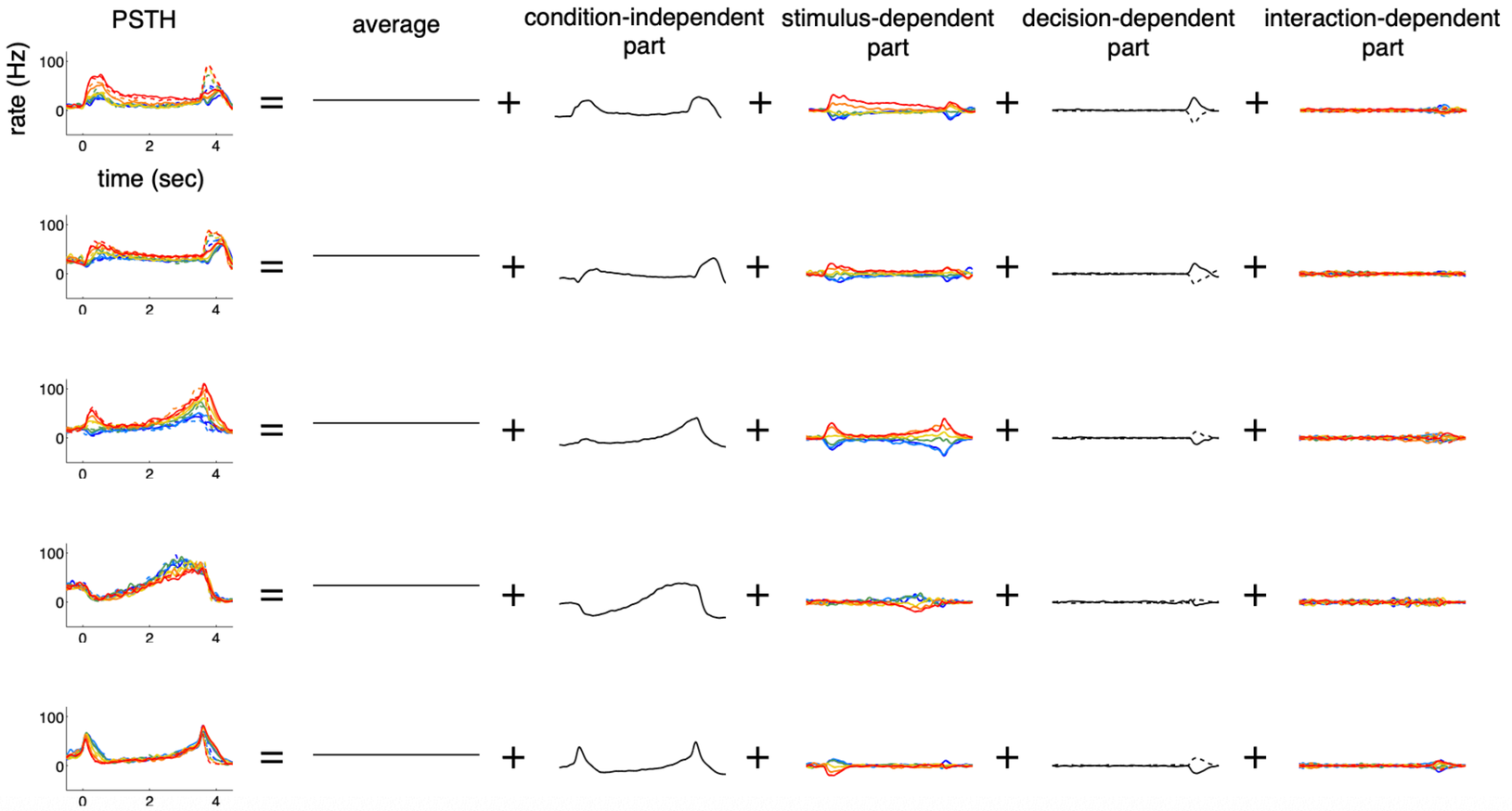
Yes No

Kobak et al. *eLife* 2016

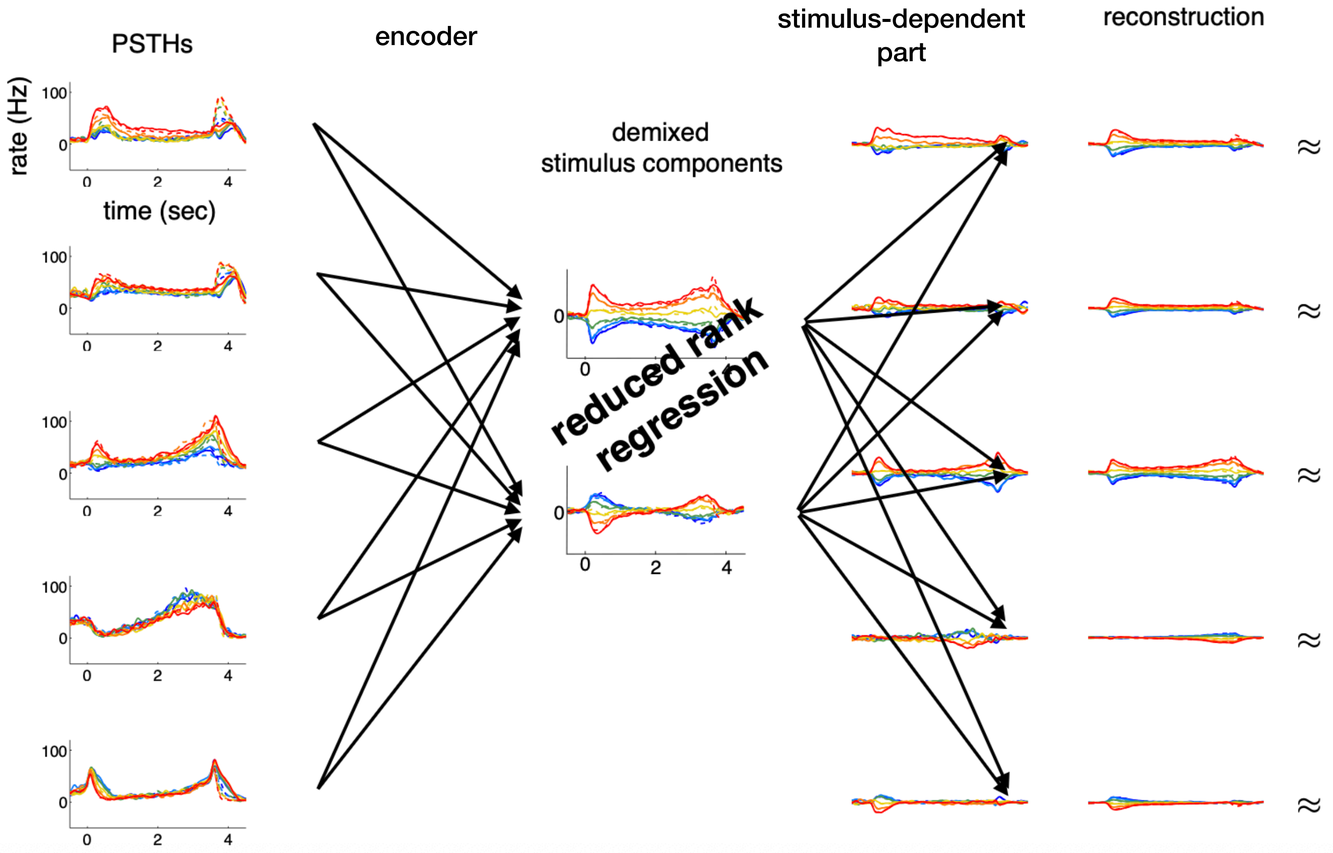
A PFC working memory task

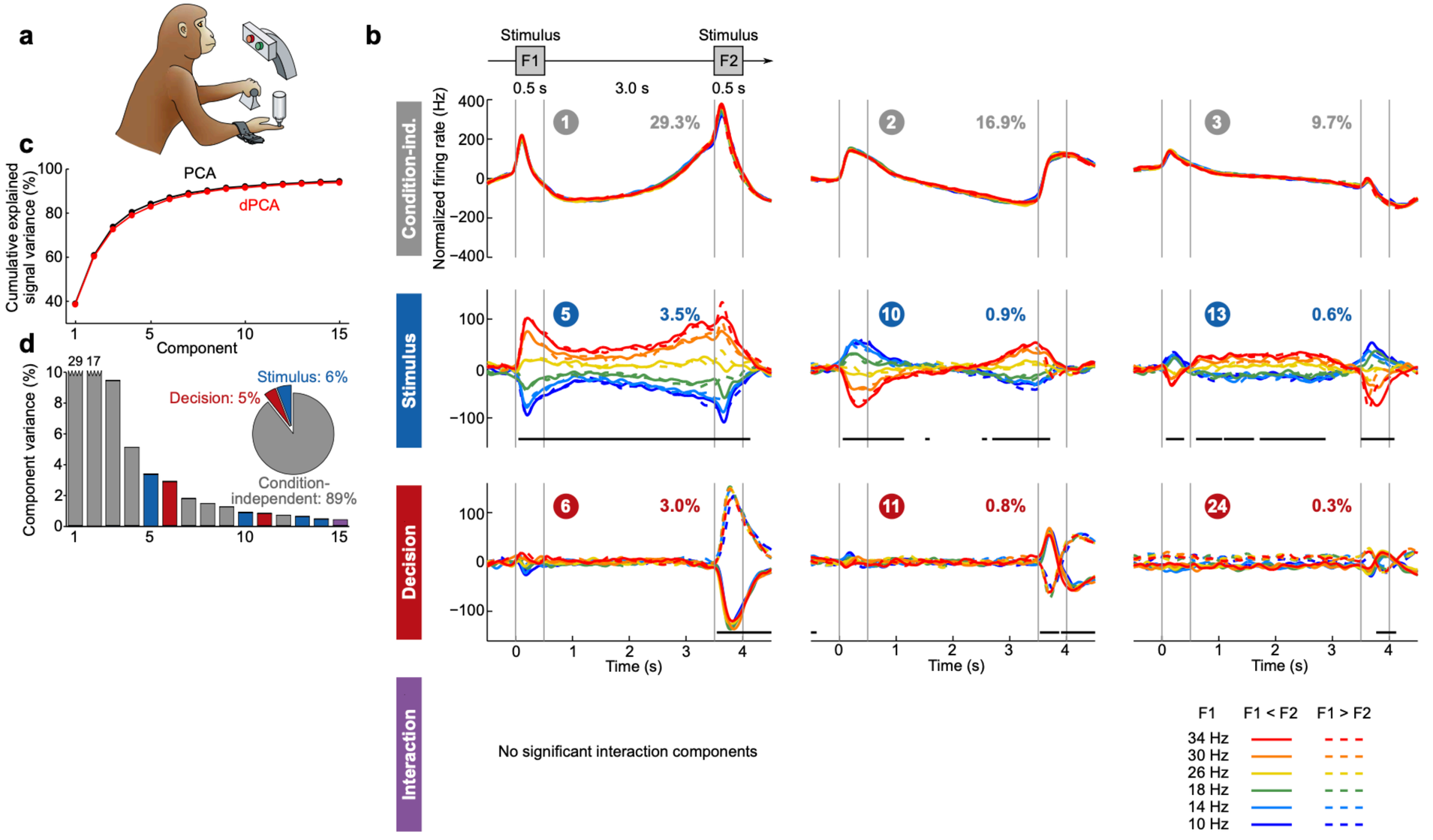


dPCA Algorithm, step 1



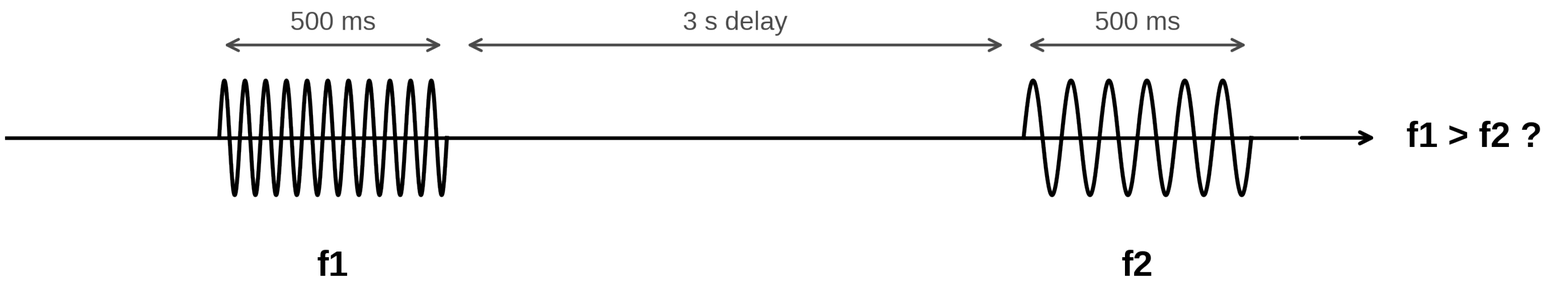
dPCA Algorithm, step 2





Kobak et al. *eLife* 2016

Exercise: dPCA on the Romo dataset



Steps

1. Load romo_allpsth.mat
2. Read the help for dpca.m
3. Set the parameters, separating time from condition
4. Explained variance with dpca_explainedVariance
5. Plot with dpca_plot

Advanced

1. Reorganize the data as neurons × stimuli × decisions × time
2. Re-run dPCA on the full factorial design

Communication subspace

What does area A send to area B?

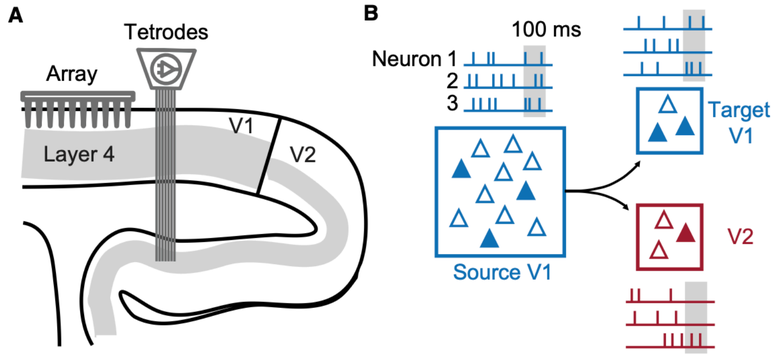
Setup

Two simultaneously recorded populations, both row-centered.

$$X \in \mathbb{R}^{N_X \times T}, Y \in \mathbb{R}^{N_Y \times T}$$

$$B_{OLS} = YX^T(XX^\top)^{-1}$$

$$\hat{Y} = B_{OLS}X = YP$$

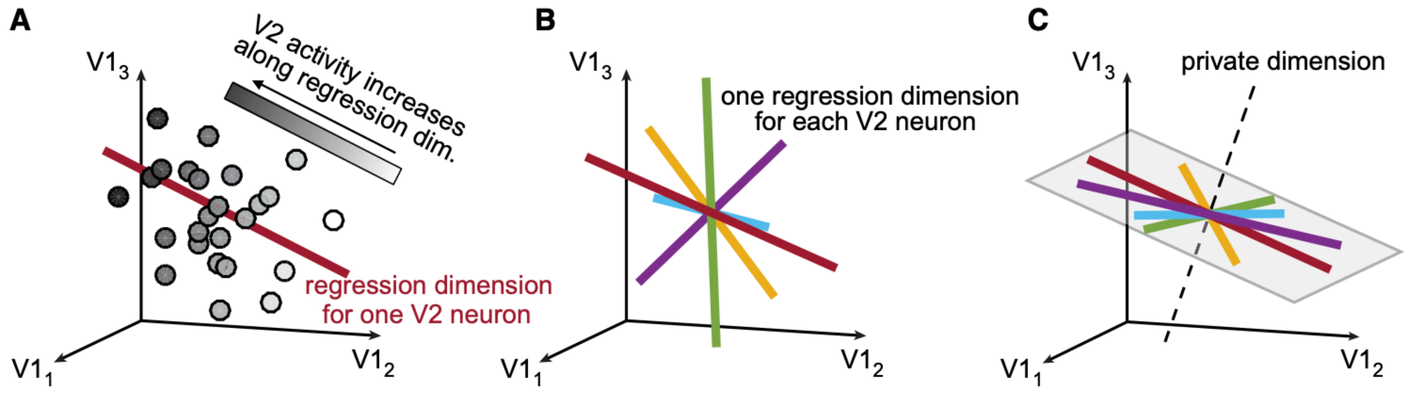


$\hat{Y}$: the best prediction for target from source

Semedo et al., 2019

Is the interaction high or low dimensional?

Again, reduced-rank regression!



$$\min_{\text{rank } B \leq r} \left\| \hat{Y} - BX \right\|_F^2$$

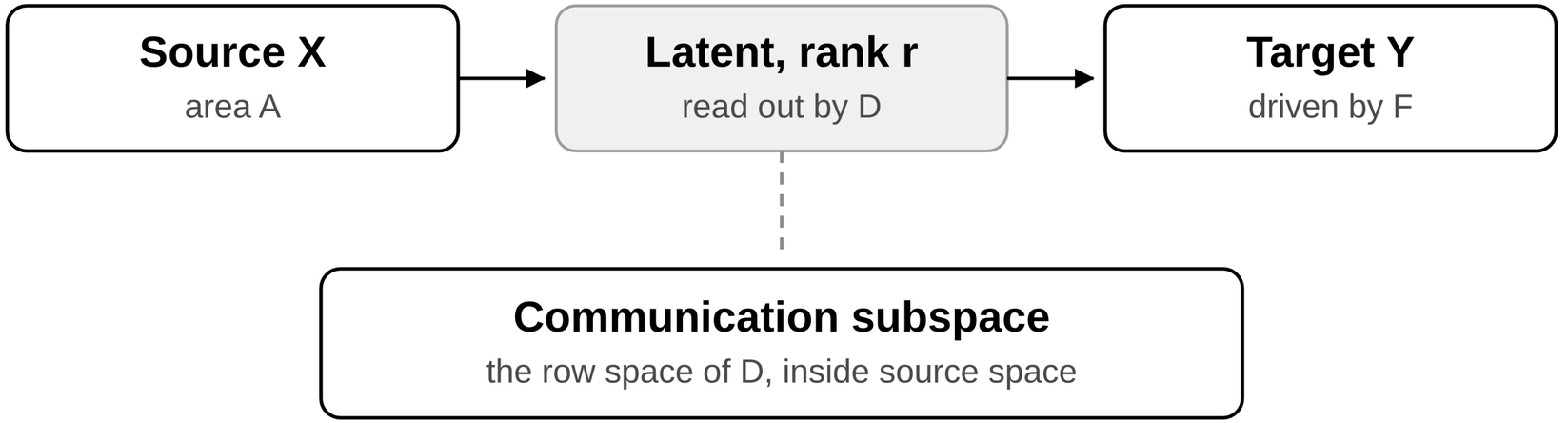
$$\hat{Y} = U \Sigma V^\top$$

$$BX = U_r U_r^\top B_{OLS} X$$

$$B = U_r U_r^\top B_{OLS}$$

Two spaces of $B \in \mathbb{R}^{N_Y \times N_x}$

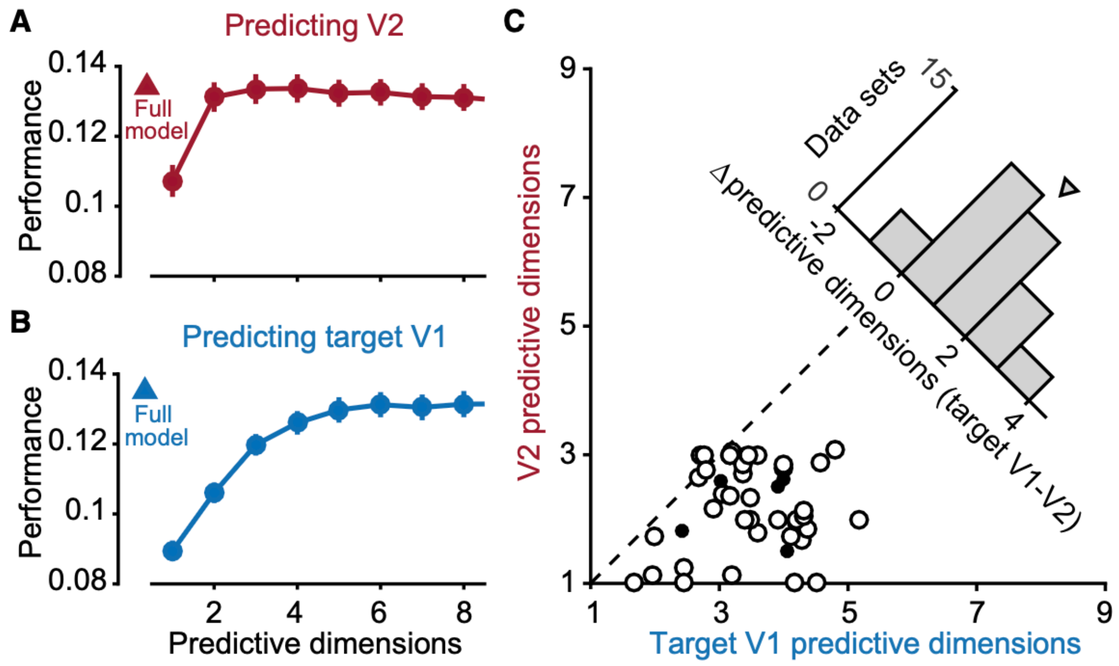
$$B = U_r U_r^\top B_{OLS} \qquad D = U_r^\top B_{OLS} \in \mathbb{R}^{r \times N_X} \qquad F = U_r \in \mathbb{R}^{N_Y \times r}$$



- Row space in source space: which dimensions are read out.
- Column space in target space: which dimensions get driven.
- $\text{null}(D) =$ private subspace (does not involve in the interaction)

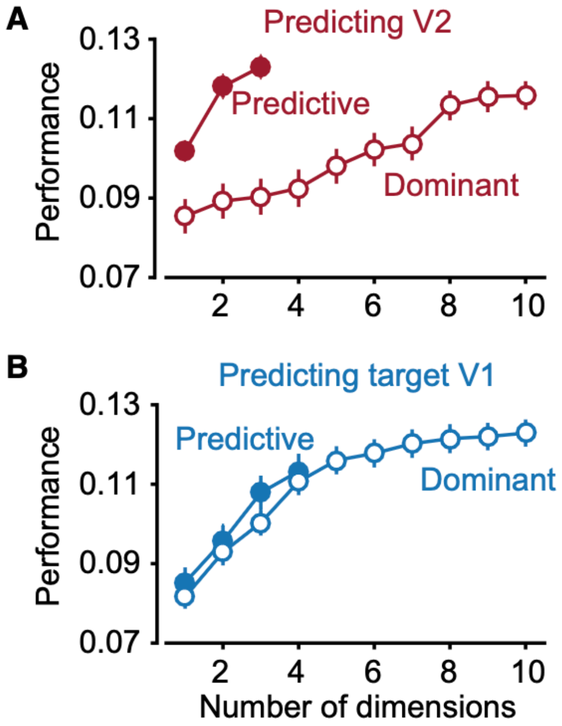
Low dimensional interaction between V1 and V2

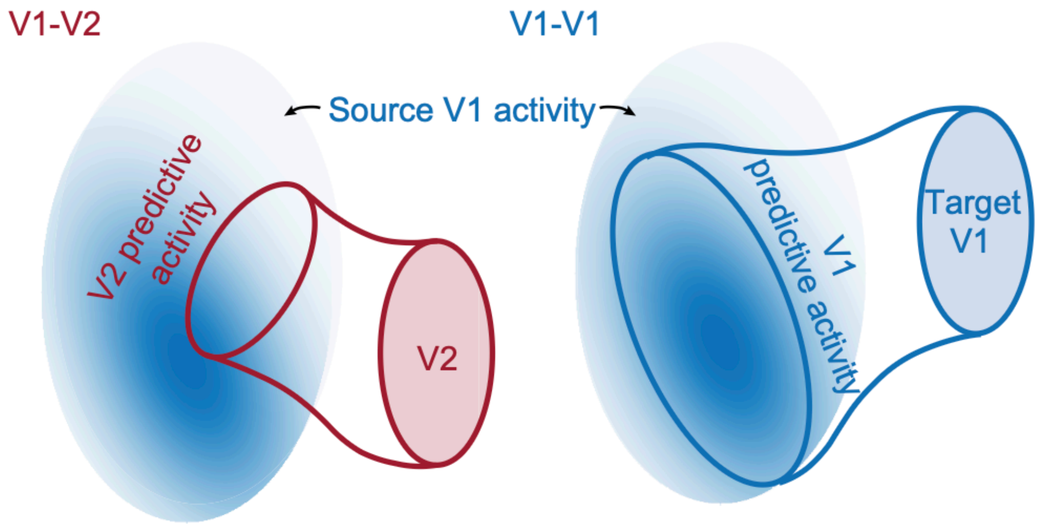
$r \approx 2$



Semedo et al., 2019

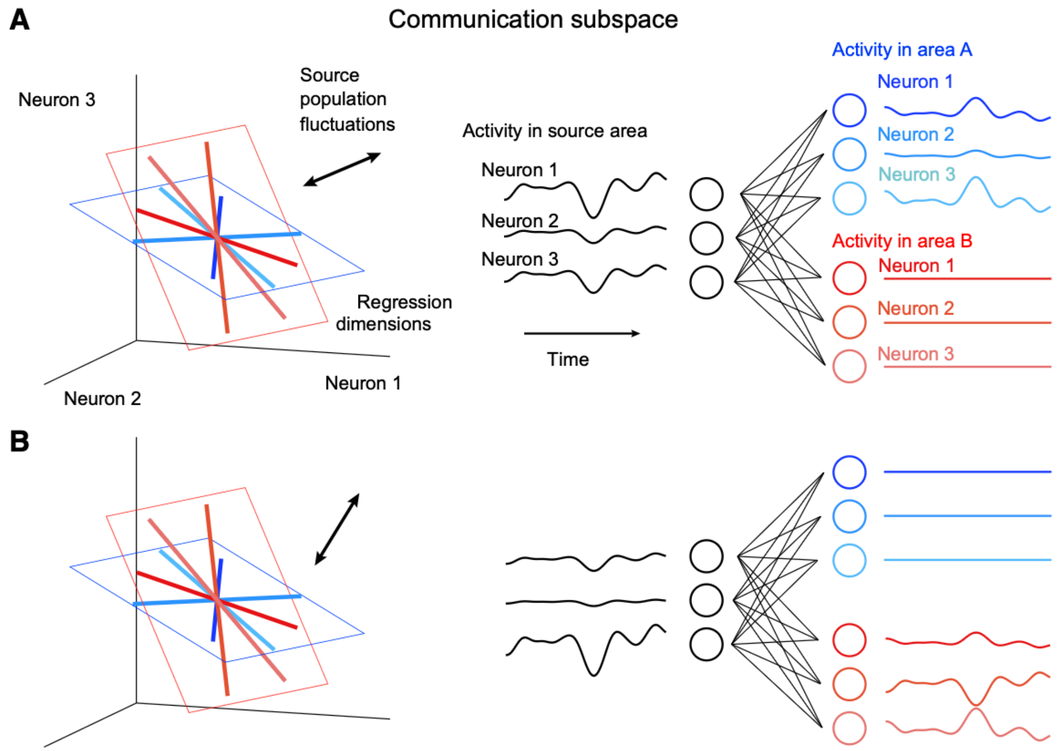
The predictive dimensions for V2 is not the dominant dimensions in V1





Semedo et al., 2019

Selective communication between different areas



Semedo et al., 2019

Limitations

- Not causal. Common input to both areas produces exactly the same signature as directed communication.
- Instantaneous by default. Time-lagged RRR is the natural extension.
- Linear. What about nonlinear mapping?

Regress on the right.

Truncate on the left.

Change what sits on the left-hand side and the same estimator becomes PCA, demixed PCA, or a communication subspace.